

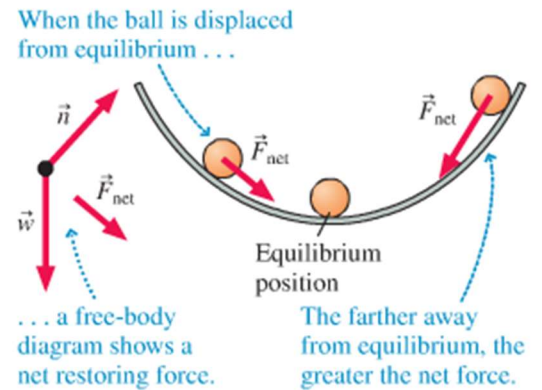
## WAVES AND SOUND

### Equilibrium and Oscillations

- **Equilibrium position** → where an oscillating object will come to rest with no net force acting on it.
- **Restoring force** → a force that acts to restore equilibrium.
  - Ex: gravity on a pendulum
- **Oscillation** → any repetitive motion.
  - Ex: pendulums, masses bobbing on springs, waves
- **Period** → the time it takes to complete *one* full cycle.
  - Unit: seconds (s)
- **Frequency** → The number of cycles per second.
  - Units: Hertz (Hz)
  - 1 Hz = 1 cycle/s = 1 s<sup>-1</sup>

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

**FIGURE 14.1** Equilibrium and restoring forces for a ball in a bowl.



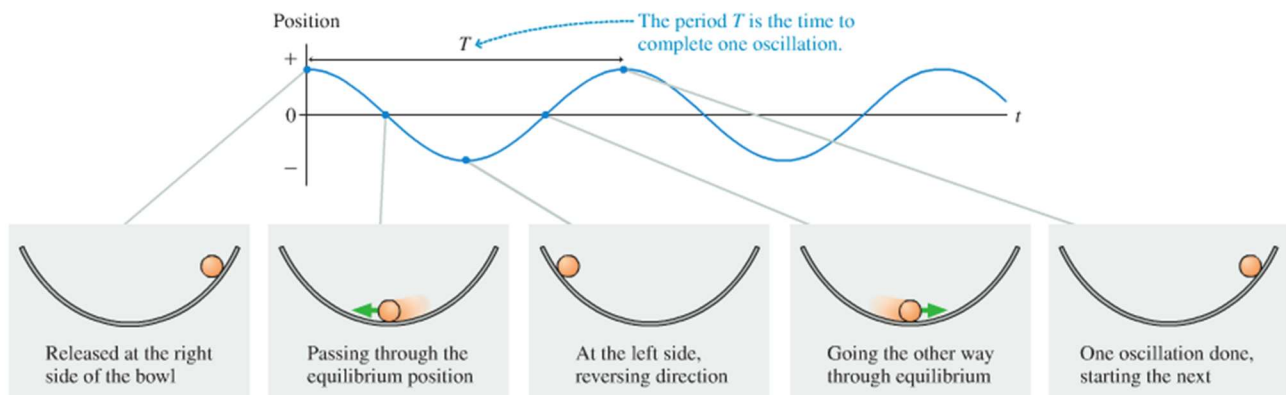
### **EXAMPLE 14.1** Frequency and period of a radio station

An FM radio station broadcasts an oscillating radio wave at a frequency of 100 MHz. What is the period of the oscillation?

### Simple Harmonic Motion

- **Sinusoidal** → A graph or function that has the shape of a sine or cosine function.
- **Simple Harmonic Motion (SHM)** → a sinusoidal oscillation.
  - Ex: a pendulum

**FIGURE 14.4** Constructing a position-versus-time graph for a marble rolling in a bowl.

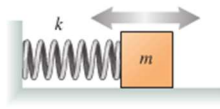


- SHM is very common, but almost all cases can be represented as either a mass bobbing on a spring or a pendulum.
- The marble in a bowl acts just like a pendulum swinging.

## Examples of simple harmonic motion

### Oscillating system

#### Mass on a spring



The mass oscillates back and forth due to the restoring force of the spring. The period depends on the mass and the stiffness of the spring.

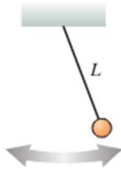
### Related real-world example BIO

#### Vibrations in the ear



Sound waves entering the ear cause the oscillation of a membrane in the cochlea. The vibration can be modeled as a mass on a spring. The period of oscillation of a segment of the membrane depends on mass (the thickness of the membrane) and stiffness (the rigidity of the membrane).

#### Pendulum



The mass oscillates back and forth due to the restoring gravitational force. The period depends on the length of the pendulum and the free-fall acceleration  $g$ .

#### Motion of legs while walking



The motion of a walking animal's legs can be modeled as pendulum motion. The rate at which the legs swing depends on the length of the legs and the free-fall acceleration  $g$ .

### STOP TO THINK 14.1

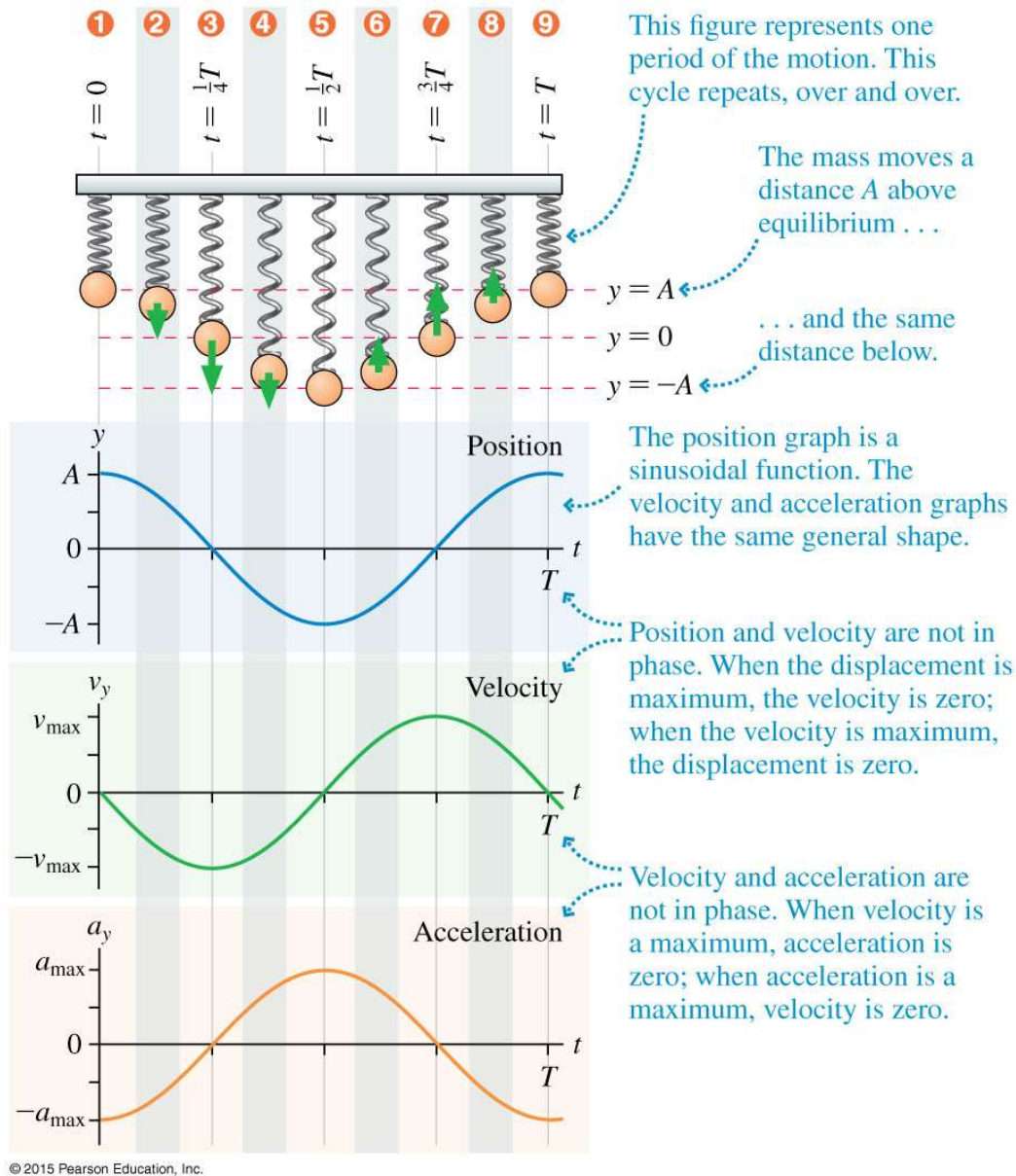
Two oscillating systems have periods  $T_1$  and  $T_2$ , with  $T_1 < T_2$ . How are the frequencies of the two systems related?

A.  $f_1 < f_2$

B.  $f_1 = f_2$

C.  $f_1 > f_2$

- **Amplitude**  $\rightarrow$  the maximum displacement from an object's equilibrium position.
  - $\triangleright$  The object will oscillate between  $x = -A$  and  $x = A$ .



**STOP TO THINK 14.3** The graphs in the table above apply to pendulum motion as well as the motion of a mass on a spring. A pendulum in a clock has a period of 2.0 seconds. You pull the pendulum to the right—a positive displacement—and let it go; we call this time  $t = 0$  s. At what time will the pendulum (a) first be at its maximum negative displacement, (b) first have its maximum speed, (c) first have its maximum positive velocity, and (d) first have its maximum positive acceleration?

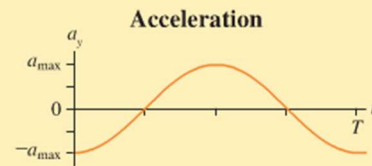
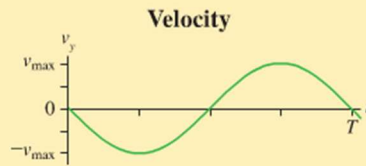
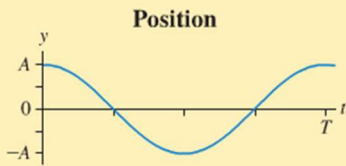
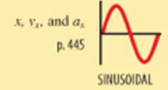
- A. 0.5 s      B. 1.0 s      C. 1.5 s      D. 2.0 s

### EXAMPLE 14.2 Motion of a glider on a spring

An air-track glider oscillates horizontally on a spring at a frequency of 0.50 Hz. Suppose the glider is pulled to the right of its equilibrium position by 12 cm and then released. Where will the glider be 1.0 s after its release? What is its velocity at this point?

#### SYNTHESIS 14.1 Describing simple harmonic motion

The position, velocity, and acceleration of objects undergoing simple harmonic motion are related sinusoidal functions.



At time  $t$ , the displacement, velocity and acceleration are given by:

$$x(t) = A \cos(2\pi ft)$$

$$v_x(t) = -v_{\max} \sin(2\pi ft)$$

$$a_x(t) = -a_{\max} \cos(2\pi ft)$$

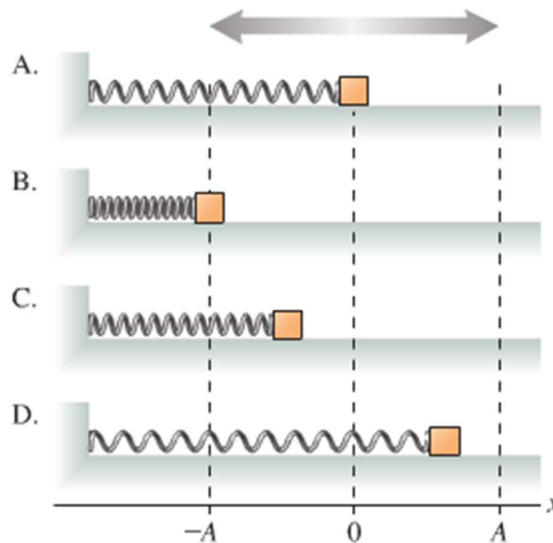
The maximum values of the displacement, velocity, and acceleration are determined by the amplitude  $A$  and the frequency  $f$ :

$$x_{\max} = A$$

$$v_{\max} = 2\pi fA$$

$$a_{\max} = (2\pi f)^2 A$$

**STOP TO THINK 14.4** The figures show four identical oscillators at different points in their motion. Which is moving fastest at the time shown?



- The period of an object oscillating on a spring can be found using

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

- The period of a pendulum can be found using

$$T_p = 2\pi \sqrt{\frac{l}{g}}$$

- Note that the period and frequency of something in SHM does not depend on the amplitude.
  - I.e., how far up you release your pendulum does NOT make a difference.

#### CONCEPTUAL EXAMPLE 14.6

#### Changing mass, changing period

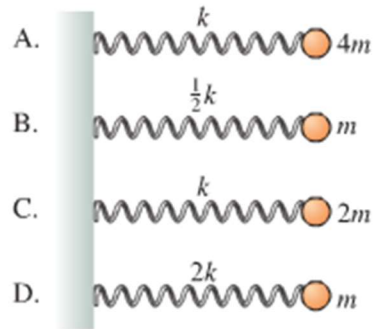
An astronaut measures her mass each day using the Body Mass Measurement Device, as described at left. During an 8-day flight, her mass steadily decreases. How does this change the frequency of her oscillatory motion on the device?



**Measuring mass in space** Astronauts on extended space flights monitor their mass to track the effects of weightlessness on their bodies. But because they are weightless, they can't just hop on a scale! Instead, they use an ingenious device in which an astronaut sitting on a platform oscillates back and forth due to the restoring force of a spring. The astronaut is the moving mass in a mass-spring system, so a measurement of the period of the motion allows a determination of an astronaut's mass.

**STOP TO THINK 14.5**

Four mass-spring systems have masses and spring constants shown here. Rank in order, from highest to lowest, the frequencies of the oscillations.

**EXAMPLE 14.10****Designing a pendulum for a clock**

A grandfather clock is designed so that one swing of the pendulum in either direction takes 1.00 s. What is the length of the pendulum?

**EXAMPLE 14.11****Finding the frequency of a swinging leg** BIO

A student in a biomechanics lab measures the length of his leg, inside a shoe, from hip to heel, to be 0.90 m. What is the frequency of the pendulum motion of the student's leg? What is the period?

- **Damping** → the tendency of oscillations to decrease over time due to drag (resistance).

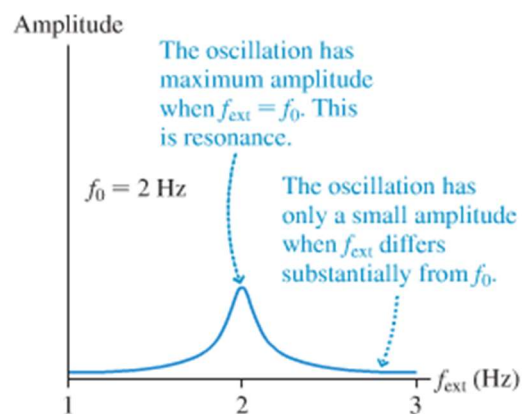
**RESONANCE**

- If you jiggle a cup of water, the water sloshes back and forth.
  - The water is an *oscillator* being subjected to a periodic external force (from your hand).
  - This is an example of **driven oscillation**.
  - Other examples: earthquakes forcing the ground to oscillate, etc.
- When an oscillating system is free from external forces, it will oscillate at a certain frequency or sets of frequencies. This is called the **natural frequency** of the oscillator.
- A periodic external force with some **driving frequency** may act on an oscillating system.
- This driving force will cause the system to oscillate at the driving frequency.
- **Resonance** → when the driving frequency is the same as the natural frequency.
  - Results in oscillations with large amplitude (high energy).



**Serious sloshing** Water in Canada's Bay of Fundy oscillates back and forth with a period of 12 hours—nearly equal to the period of the moon's tidal force, which gives two daily high tides 12.5 hours apart. This *resonance*, a close match between the bay's natural frequency and the moon's driving frequency, produces a huge tidal amplitude. Low tide can be 16 m below high tide, leaving boats high and dry.

**FIGURE 14.24** The response curve shows the amplitude of a driven oscillator at frequencies near its natural frequency  $f_0 = 2$  Hz.



### CONCEPTUAL EXAMPLE 14.13 Fixing an unwanted resonance

Railroad cars have a natural frequency at which they rock side to side. This can lead to problems on certain stretches of track that have bumps where the rails join. If the joints alternate sides, with a bump on the left rail and then on the right, a train car moving down the track is bumped one way and then the other. In some cases, bumps have caused rocking with amplitude large enough to derail the train. A train moving down the track at a certain speed is experiencing a large amplitude of oscillation due to alternating joints in the track. How can the driver correct this potentially dangerous situation?

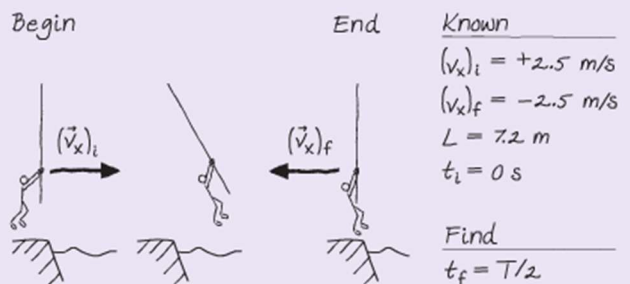
### INTEGRATED EXAMPLE 14.14 Analyzing the motion of a rope swing

A rope hangs down from a high tree branch at the edge of a river. Josh, who has mass 65 kg, trots at 2.5 m/s to the edge of the river, grabs the rope 7.2 m below where it is tied to the branch, and swings out over the river.

- What is the minimum time that Josh must hang on to make it back to shore?
- What is the maximum tension in the rope?

**PREPARE** This is pendulum motion, with Josh as the mass at the end of the string, the 7.2 m length of rope. Josh grabs the rope, swings out, and comes back to shore, as shown in FIGURE 14.28. As he swings out, his displacement increases and his speed decreases.

FIGURE 14.28 Pendulum motion of a person swinging on a rope.

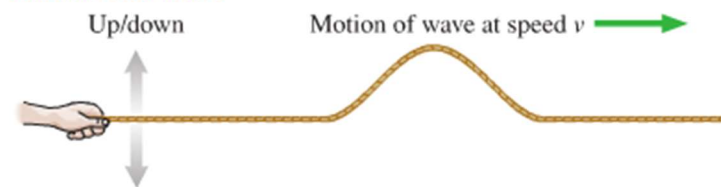


## MECHANICAL WAVES

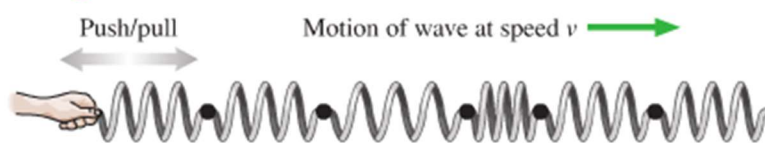
- **Mechanical Wave**  $\rightarrow$  waves that require a medium through which to move.
  - $\triangleright$  As the wave passes through the medium, the atoms that make up the medium are displaced from equilibrium position.
  - $\triangleright$  i.e., the medium experiences a **disturbance**.
- Disturbance of a medium *always* has a source. (Conservation of Energy! No magic!)
- Once created the disturbance travels through the medium at some **wave speed**,  $v$ .
- **A wave transfers energy. The particles of the medium itself do not travel.**
- **Transverse waves**  $\rightarrow$  the medium moves perpendicular to the direction of energy flow.
- **Longitudinal waves**  $\rightarrow$  the medium moves parallel to the direction of energy flow.

### Two types of wave motion

#### A transverse wave



#### A longitudinal wave



**STOP TO THINK 15.1** Spectators at a sporting event do “The Wave,” as shown in the photo on the preceding page. Is this a transverse or longitudinal wave?

### WAVES ON A STRING

- Waves on a string are transverse.
- The wave speed does not depend on the size of the pulse/disturbance or how it was generated. It depends *only* on the properties of the medium.
- **Linear density**  $\rightarrow$  mass to length ration of a string.

$$\mu = \frac{m}{L}$$

- A string with a greater tension responds more rapidly, so as tension increases, so does wave speed.
- A string with greater linear density has greater inertia, so as linear density increases, wave speed decreases.

$$v_{string} = \sqrt{\frac{T}{\mu}}$$

(T in this case is tension)

#### **EXAMPLE 15.1** When does the spider sense its lunch? BIO

All spiders are very sensitive to vibrations. An orb spider will sit at the center of its large, circular web and monitor radial threads for vibrations created when an insect lands. Assume that these threads are made of silk with a linear density of  $1.0 \times 10^{-5}$  kg/m under a tension of 0.40 N, both typical numbers. If an insect lands in the web 30 cm from the spider, how long will it take for the spider to find out?

**STOP TO THINK 15.2** Suppose you shake the end of a stretched string to produce a wave. Which of the following actions would increase the speed of the wave down the string? There may be more than one correct answer; if so, give all that are correct.

- Move your hand up and down more quickly as you generate the wave.
- Move your hand up and down a greater distance as you generate the wave.
- Use a heavier string of the same length, under the same tension.
- Use a lighter string of the same length, under the same tension.
- Stretch the string tighter to increase the tension.
- Loosen the string to decrease the tension.



## THE WAVE EQUATION

- Sinusoidal waves move one wavelength in one period, which gives us the wave equation.

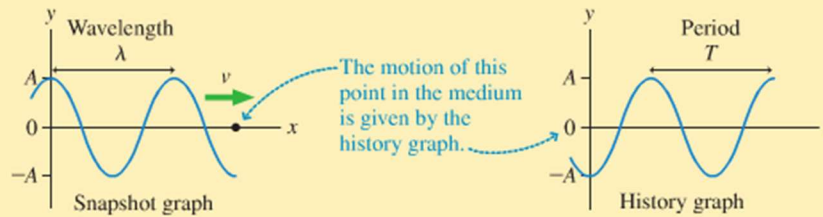
$$v = f\lambda$$

A sinusoidal wave moves one wavelength in one period, giving the fundamental relationship:

$$v = \lambda f = \frac{\lambda}{T}$$

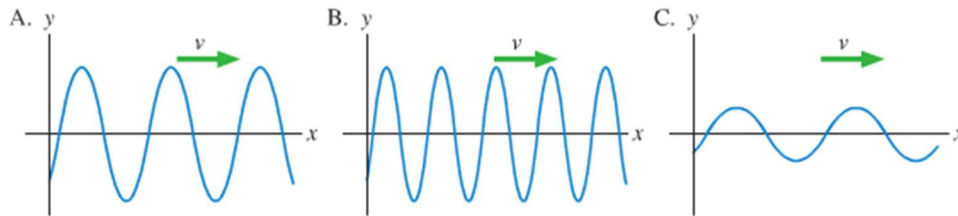
Speed of the wave (m/s)  $\rightarrow$   $v = \lambda f = \frac{\lambda}{T}$   $\leftarrow$  Wavelength (m)  
 Frequency of the wave (Hz)  $\leftarrow$   $f$   $\leftarrow$  Period of the wave (s)

As a sinusoidal wave travels, each point in the medium moves with simple harmonic motion.



### STOP TO THINK 15.4

Three waves travel to the right with the same speed. Which wave has the highest frequency? All three graphs have the same horizontal scale.

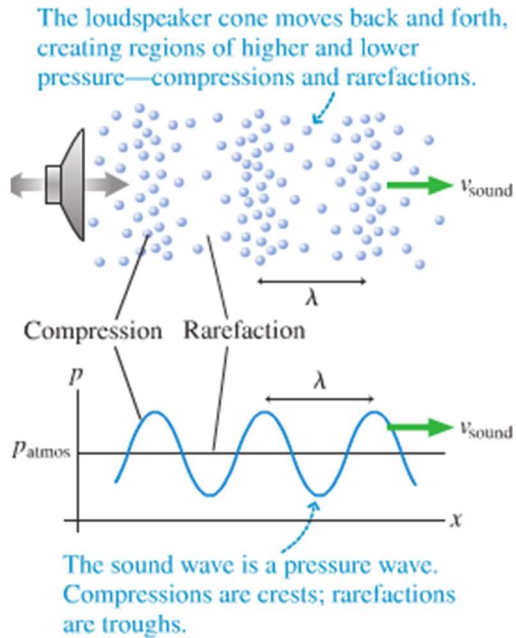


Example:

Ocean waves are observed to travel along the water surface during a developing storm. A Coast Guard weather station observes that there is a vertical distance from high point to low point of 4.6 meters and a horizontal distance of 8.6 meters between adjacent crests. The waves splash into the station once every 6.2 seconds. Determine the frequency and the speed of these waves.

## SOUND

- Sound is a pressure wave.
  - Compressions  $\rightarrow$  areas of high pressure
  - Rarefactions  $\rightarrow$  areas of low pressure.



- Range of human hearing: 20 Hz – 20,000 Hz
  - **Infrasound** → anything below 20 Hz
  - **Ultrasound** → anything above 20,000 Hz

#### EXAMPLE 15.6 Range of wavelengths of sound

What are the wavelengths of sound waves at the limits of human hearing and at the midrange frequency of 500 Hz? Notes sung by human voices are near 500 Hz, as are notes played by striking keys near the center of a piano keyboard.

#### EXAMPLE 15.7 Ultrasonic frequencies in medicine **BIO**

To make a sufficiently detailed ultrasound image of a fetus in its mother's uterus, a physician has decided that a wavelength of 0.50 mm is needed. What frequency is required?

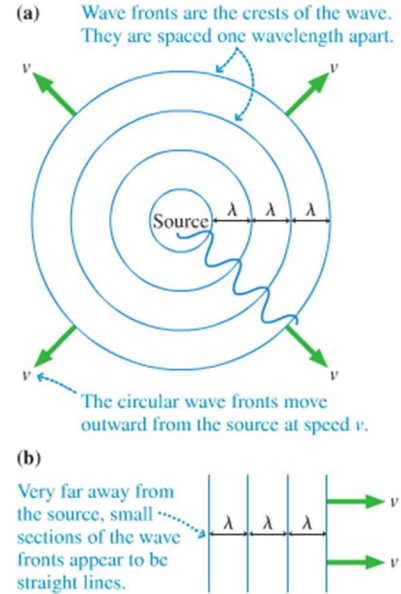
**PREPARE** The speed of ultrasound in the body is given in Table 15.1 as 1540 m/s.



Computer processing of an ultrasound image shows fine detail.

## ENERGY AND INTENSITY

- Imagine a pebble dropped in a pond. The resulting waves would look like image (a) to the right.
  - **Wave fronts** → the lines that locate the crests of waves.
  - Wave fronts are separated from each other by one wavelength.
- This sort of wave is a **circular wave**.
- If you were to move far from the source of a circular wave, the curvature of the waves would be unnoticeable to you.
  - They would appear to be parallel lines, travelling at speed  $v$ , still separated by one wavelength from each other.



- **Spherical Waves** → waves that move in 3-dimensions.
  - Sound waves and light waves are spherical waves.
  - The crests of the wave form a series of spherical shells separated from each other by one wavelength.
  - You can still picture them as we did in figure (a) above, but that would represent a slice of the sphere.
- If you were to be located far from the source, the part of the wave front that you could see would be a small patch of the larger sphere.
  - If you are far enough away (i.e., a sphere of a large radius), then the curvature of the patch of wave front would be negligible and you would appear to be a plane.

FIGURE 15.15 A plane wave.

Very far from the source, small segments of spherical wave fronts appear to be planes. The wave is cresting at every point in these planes.

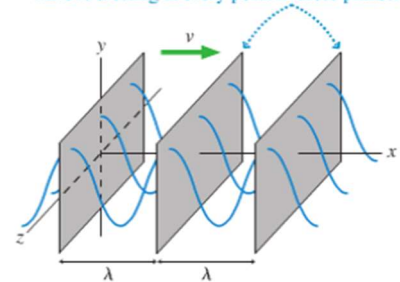
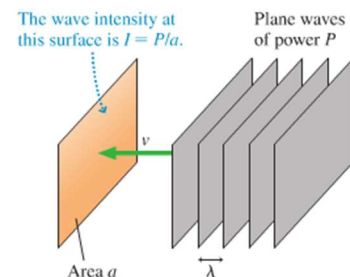


FIGURE 15.16 Plane waves of power  $P$  impinge on area  $a$ .



- **Power** → the rate in Watts (J/s) at which the wave transfers energy.
- Brightness (light) or loudness (sound) depend not only on the power of the source, but on the *area* that receives that power.
- This quantity is called **intensity (I)**.
  - Units:  $\text{W/m}^2$
  - It's a power to area ratio.

$$I = \frac{P}{A}$$

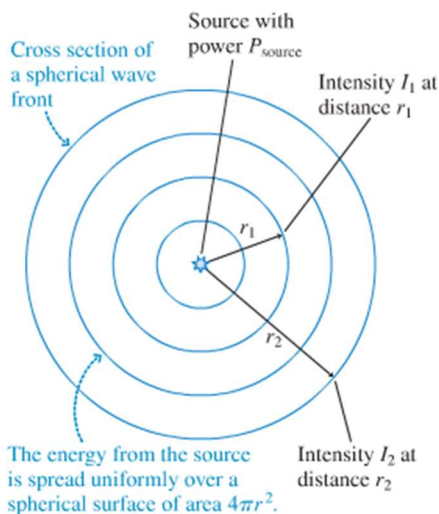
**EXAMPLE 15.9****The intensity of a laser beam**

A bright, tightly focused laser pointer emits 1.0 mW of light power into a beam that is 1.0 mm in diameter. What is the intensity of the laser beam?

- Sound and light become less intense as you move further from the source because the spherical waves spread out to fill larger and larger volumes of space.
- If a source of spherical waves radiates uniformly in all directions, then the power at distance  $r$  is spread uniformly over the surface of the sphere at radius  $r$ .

$$I = \frac{P_{\text{source}}}{4\pi r^2}$$

- The energy per area must decrease in proportion to the surface area of a sphere.



**The better to hear you with** **BIO** The great grey owl has its ears on the front of its face, hidden behind its facial feathers. Its round face works like a radar dish, collecting the energy of sound waves over a large area and “funneling” it into the ears. This allows owls to sense very quiet sounds.

- If you want to compare the intensity at two points located at different radii from the power source:

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

**EXAMPLE 15.10 Intensity of sunlight on Mars**

The intensity of sunlight on the earth's surface is approximately  $1000 \text{ W/m}^2$  at noon on a summer day. Mars orbits at a distance from the sun approximately 1.5 times that of earth.

- Assuming similar absorption of energy by the Martian atmosphere, what would you predict for the intensity of sunlight at noon during the Martian summer?
- The Sojourner rover, an early Mars rover, had a rectangular array of solar cells approximately 0.60 m long and 0.37 m wide. What is the maximum solar energy this array could capture?
- If we assume a solar-to-electric conversion efficiency of 18%, typical of high-quality solar cells, what is the maximum useful power the solar cells could produce?

**STOP TO THINK 15.6** A plane wave, a circular wave, and a spherical wave all have the same intensity. Each of the waves travels the same distance. Afterward, which wave has the highest intensity?

- A. The plane wave    B. The circular wave    C. The spherical wave

**LOUDNESS OF SOUND**

- **Sound Intensity Level** → a measurement of the “loudness” of sound.
- Because the range of intensities (in  $\text{W/m}^2$ ) the human ear can detect is so large, we often use a logarithmic scale.
  - Called the **decibel scale**.
  - Average value for the lowest sound able to be heard is  $1 \times 10^{-12} \text{ W/m}^2 \rightarrow 0 \text{ dB}$
  - Called the threshold of hearing.

Source	Intensity	Intensity Level	# of Times Greater Than TOH
Threshold of Hearing (TOH)	$1 \times 10^{-12} \text{ W/m}^2$	0 dB	$10^0$
Rustling Leaves	$1 \times 10^{-11} \text{ W/m}^2$	10 dB	$10^1$
Whisper	$1 \times 10^{-10} \text{ W/m}^2$	20 dB	$10^2$
Normal Conversation	$1 \times 10^{-6} \text{ W/m}^2$	60 dB	$10^6$
Busy Street Traffic	$1 \times 10^{-5} \text{ W/m}^2$	70 dB	$10^7$
Vacuum Cleaner	$1 \times 10^{-4} \text{ W/m}^2$	80 dB	$10^8$
Large Orchestra	$6.3 \times 10^{-3} \text{ W/m}^2$	98 dB	$10^{9.8}$
Walkman at Maximum Level	$1 \times 10^{-2} \text{ W/m}^2$	100 dB	$10^{10}$
Front Rows of Rock Concert	$1 \times 10^{-1} \text{ W/m}^2$	110 dB	$10^{11}$
Threshold of Pain	$1 \times 10^1 \text{ W/m}^2$	130 dB	$10^{13}$
Military Jet Takeoff	$1 \times 10^2 \text{ W/m}^2$	140 dB	$10^{14}$
Instant Perforation of Eardrum	$1 \times 10^4 \text{ W/m}^2$	160 dB	$10^{16}$

$$\beta = (10 \text{ dB}) \log_{10} \left( \frac{I}{I_0} \right)$$

-  $I_0 \rightarrow 1.0 \times 10^{-12} \text{ W/m}^2$

### EXAMPLE 15.11 Finding the loudness of a shout

A person shouting at the top of his lungs emits about 1.0 W of energy as sound waves. What is the sound intensity level 1.0 m from such a person?

### EXAMPLE 15.12 How far away can you hear a conversation?

The sound intensity level 1.0 m from a person talking in a normal conversational voice is 60 dB. Suppose you are outside, 100 m from the person speaking. If it is a very quiet day with minimal background noise, will you be able to hear him or her?

**SOLVE** T is 60 dB.

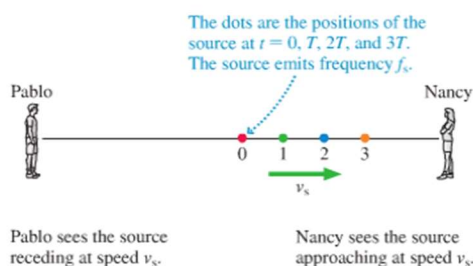
**STOP TO THINK 15.7** You are overhearing a very heated conversation that registers 80 dB. You walk some distance away so that the intensity decreases by a factor of 100. What is the sound intensity level now?

- A. 70 dB    B. 60 dB    C. 50 dB    D. 40 dB    E. 30 dB    F. 20 dB

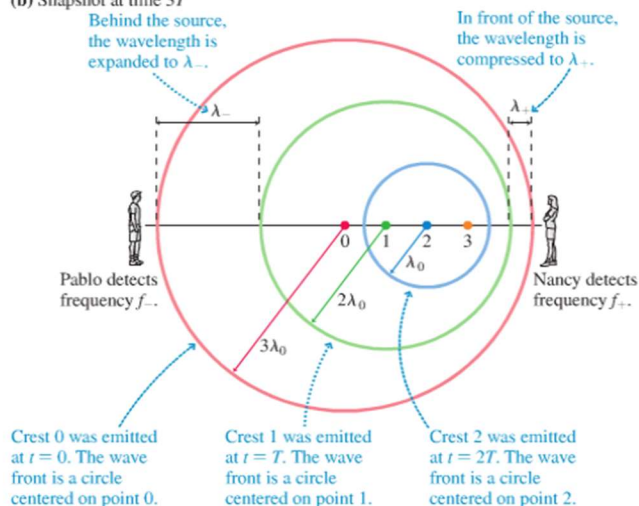
## THE DOPPLER EFFECT

- Remember that all motion is relative to the observer.
- **The Doppler Effect**  $\rightarrow$  perception of a wave's frequency and wavelength can change if there is relative motion between the observer and the source of the waves.
  - $\triangleright$  Johannes Doppler  $\rightarrow$  1842
  - $\triangleright$  Not just a sound phenomenon. Applies to all waves, including light.

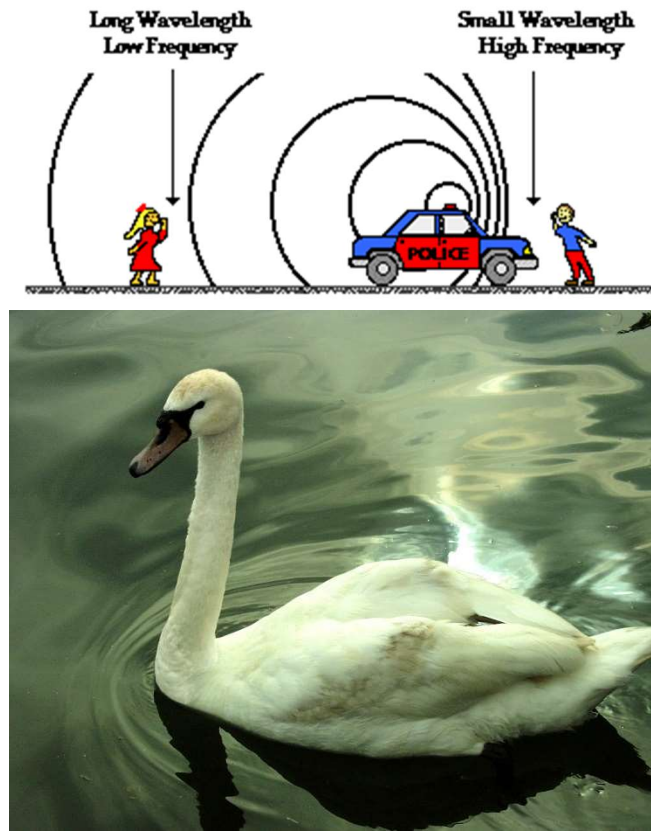
(a) Motion of the source



(b) Snapshot at time  $3T$



### The Doppler Effect for a Moving Sound Source



- As the source moves towards you, the wavelength is smaller (higher frequency).
- As the source moves away from you, the wavelength is longer (lower frequency).
- Something moving at the same velocity as the source will not hear a change in frequency.

$$f_o = f_s \left( \frac{v + v_o}{v - v_s} \right)$$

- Rules for this equation:
  - Observer moves toward source → positive velocity
  - Observer moves away from source → negative velocity
  - Source moves toward observer → positive velocity
  - Source moves away from observer → negative velocity

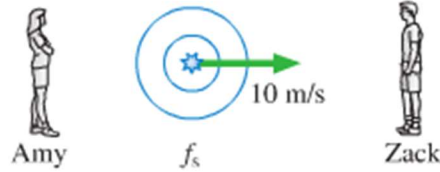
Example:

A car is travelling at 20.0 m/s (45 mi/h) and blows its horn at a constant 600 Hz. Determine the frequency heard by a stationary observer both as it approaches and recedes. Take the speed of sound to be 343 m/s.

**STOP TO THINK 15.8**

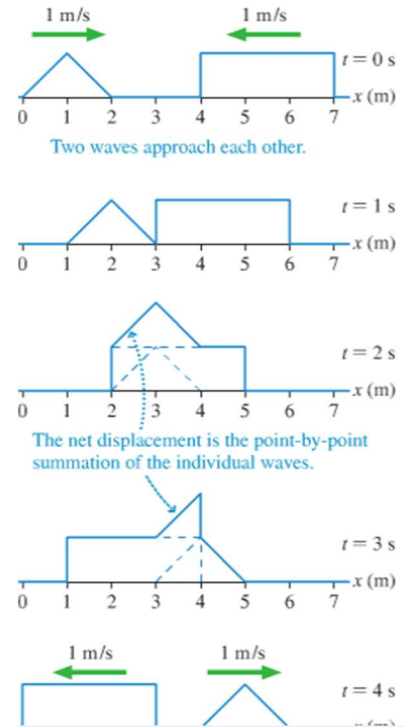
Amy and Zack are both listening to the source of sound waves that is moving to the right. Compare the frequencies each hears.

- A.  $f_{\text{Amy}} > f_{\text{Zack}}$
- B.  $f_{\text{Amy}} = f_{\text{Zack}}$
- C.  $f_{\text{Amy}} < f_{\text{Zack}}$



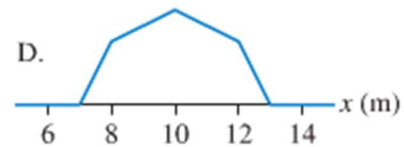
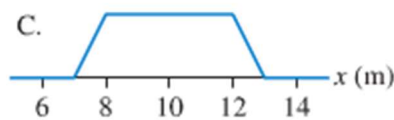
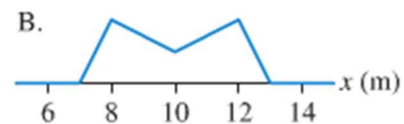
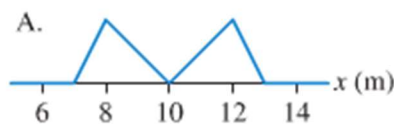
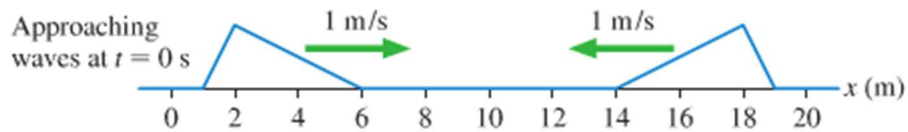
**SUPERPOSITION**

- When waves meet, there is **interference**.
  - **Constructive Interference** → leads to an increase in amplitude.
  - **Destructive Interference** → leads to a decrease in amplitude.
  - **Total Destructive Interference** → the waves cancel each other out.
- **Principle of Superposition** → When two or more waves are present simultaneously at a single point in space, the displacement of the medium at that point is the sum of their amplitudes.



**STOP TO THINK 16.1**

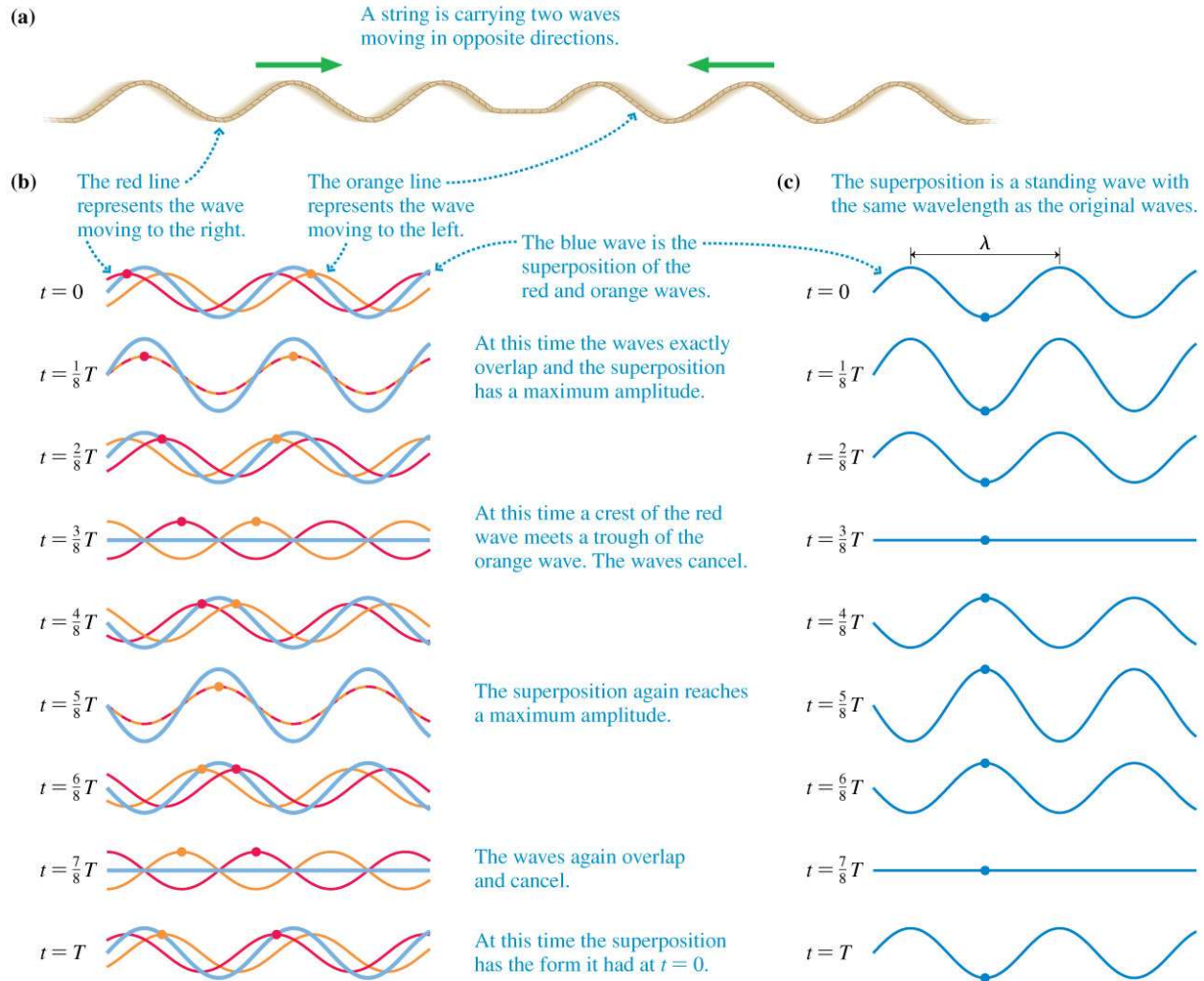
Two pulses on a string approach each other at speeds of 1 m/s. What is the shape of the string at  $t = 6$  s?





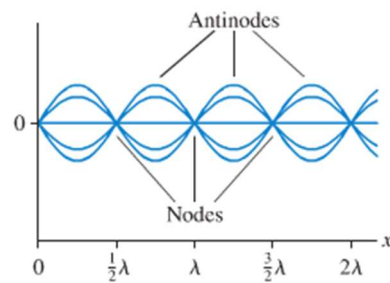
## STANDING WAVES

- When a wave reflects off of a fixed point, you can create a **standing wave**
  - Individual points in the medium oscillate, but the wave itself does not travel.
  - It's the result of two waves traveling opposite directions.



- Standing waves appear to never move.
  - Points that don't move are called **nodes** → areas of destructive interference.
  - Points that fluctuate between crests and troughs are called **antinodes** → areas of constructive interference.

**FIGURE 16.7** Superimposing multiple snapshot graphs of a standing wave clearly shows the nodes and antinodes.

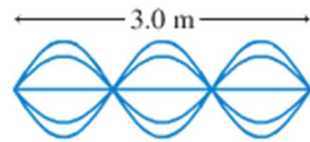


- The wavelength is twice the distance between successive nodes or successive antinodes.
- The intensity is maximum at areas of constructive interference and zero at areas of destructive interference.

**EXAMPLE 16.1** **Setting up a standing wave**

Two children hold an elastic cord at each end. Each child shakes her end of the cord 2.0 times per second, sending waves at 3.0 m/s toward the middle, where the two waves combine to create a standing wave. What is the distance between adjacent nodes?

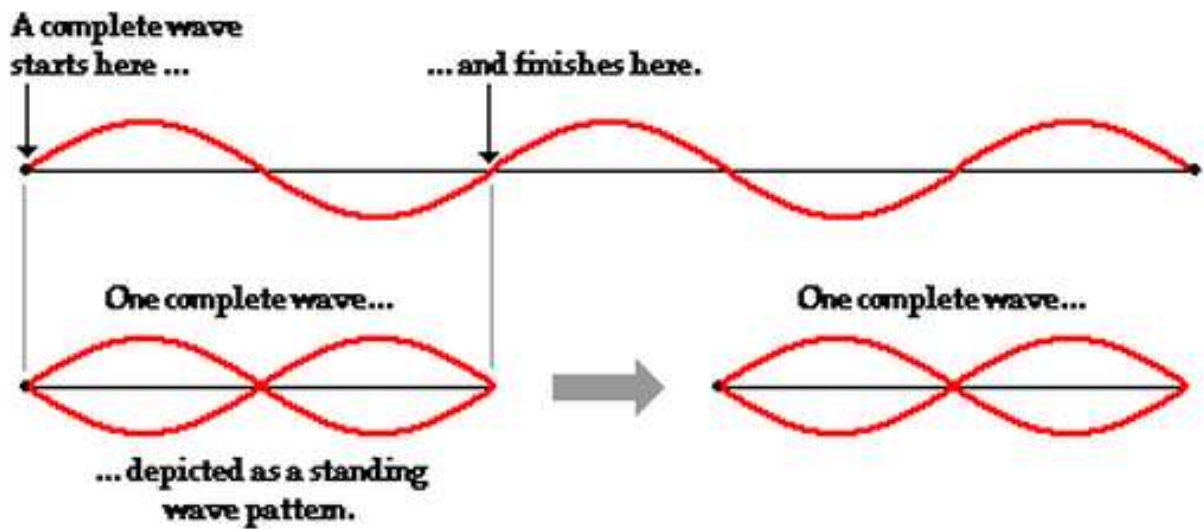
**STOP TO THINK 16.2** A standing wave is set up on a string. A series of snapshots of the wave are superimposed to produce the diagram at right. What is the wavelength?



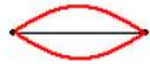
- A. 6.0 m      B. 4.0 m      C. 3.0 m      D. 2.0 m      E. 1.0 m

- The amplitude of a wave reflected is unchanged.

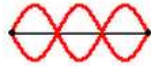
Harmonic	# of Nodes	# of Antinodes	Pattern
1st	2	1	
2nd	3	2	
3rd	4	3	
4th	5	4	
5th	6	5	
6th	7	6	
nth	n + 1	n	...



1<sup>st</sup> Harmonic:



3<sup>rd</sup> Harmonic:

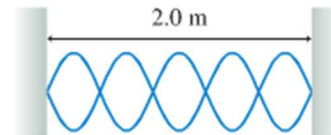


- So for the n<sup>th</sup> harmonic:

$$L = \left(\frac{n}{2}\right) \lambda$$

- n is the number of antinodes on the standing waves.
- **A standing wave can only exist on a string if the wavelength is one of the values given by the above equation.**
- n is the number of anti-nodes on the standing wave.

**STOP TO THINK 16.3** A 2.0-m-long string carries a standing wave as in the figure at right. Extend the pattern and the formulas shown in Figure 16.13 to determine the mode number and the wavelength of this particular standing-wave mode.

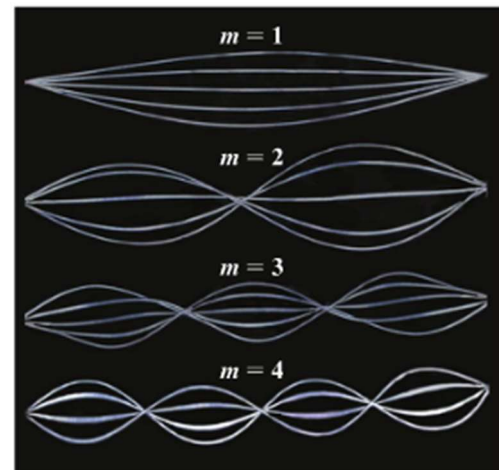


- A.  $m = 6, \lambda = 0.67 \text{ m}$       B.  $m = 6, \lambda = 0.80 \text{ m}$       C.  $m = 5, \lambda = 0.80 \text{ m}$   
D.  $m = 5, \lambda = 1.0 \text{ m}$       E.  $m = 4, \lambda = 0.80 \text{ m}$       F.  $m = 4, \lambda = 1.0 \text{ m}$

- A stretched string will support a series of standing waves as seen in the above eqn.
  - It has a series of frequencies at which it “wants” to vibrate.
  - **Resonance modes**
- The frequency at the first resonant mode can be expressed as:

$$f_1 = \frac{v}{2L}$$

- This is the **fundamental frequency** of the string.
- The other modes can be expressed in terms of the fundamental frequency.



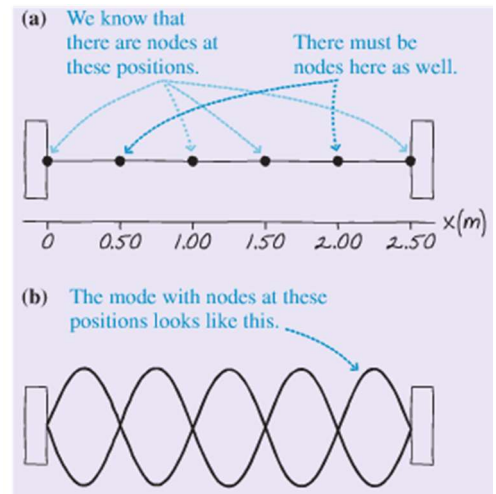
$$f_n = n f_1$$

- Where  $n = 1, 2, 3, 4, 5, \dots$

- The allowed standing wave frequencies are all whole number multiples of the fundamental frequency.
- Set of **Harmonics** → the sequence of all possible frequencies.

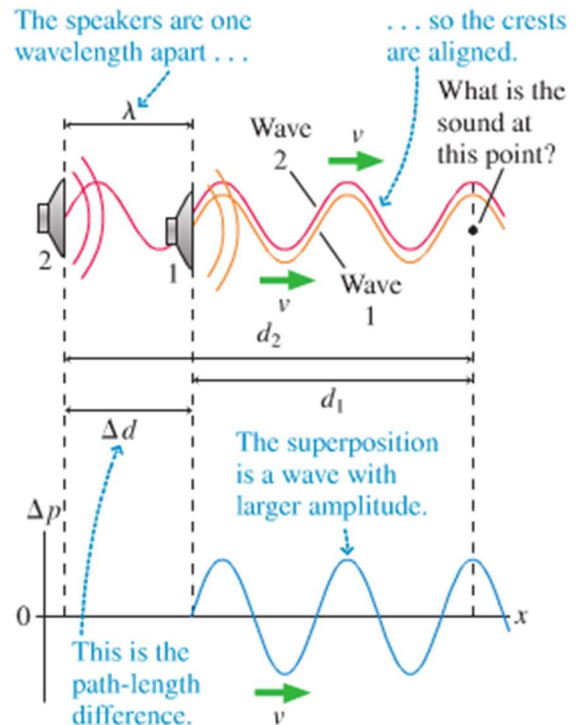
### EXAMPLE 16.2 Identifying harmonics on a string

A 2.50-m-long string vibrates as a 100 Hz standing wave with nodes at 1.00 m and 1.50 m from one end of the string and at no points in between these two. Which harmonic is this? What is the string's fundamental frequency? And what is the speed of the traveling waves on the string?

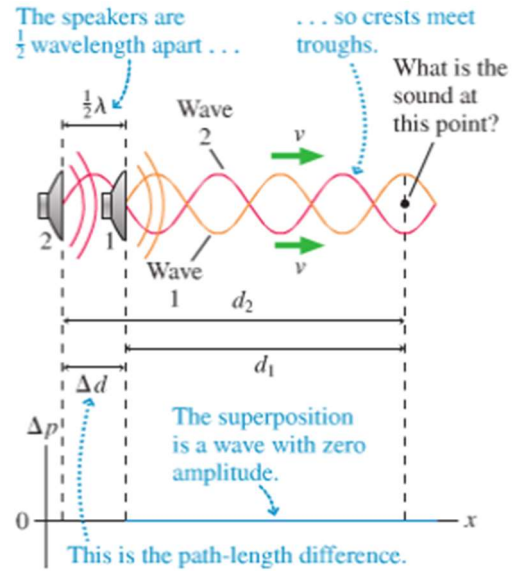


### INTERFERENCE OF WAVES FROM TWO SOURCES

- If you set up two speakers in line with each other, but separated by the wavelength of the sound wave, they are said to be **in phase**.
  - Crests are aligned with crests, troughs with troughs.
  - The superposition of their crests/troughs, leads to a wave with double the amplitude (constructive interference).
- The wave from speaker 2, however travels a further distance than the wave from speaker 1. It travels an extra distance of exactly one wavelength.
  - **Path-length difference**
- We could increase that distance by another wavelength (i.e. path-length difference =  $2\lambda$ ) and the two waves would still be in phase.
  - Same with  $3\lambda$ ,  $4\lambda$ ,  $5\lambda$ ...
  - **Two waves will be in phase and will produce constructive interference any time their path-length difference is a whole number of wave lengths.**



- If the speakers are separated by a distance of one half of a wavelength, and are **out of phase**.
- Crests meet troughs, and total destructive interference occurs. At all times, the sum of the two waves is zero.
- The same will occur at  $1.5\lambda$ ,  $2.5\lambda$ ...
- **Two waves will be out of phase and will produce destructive interference if their path-length difference is a whole number of wavelengths plus  $\frac{1}{2}$  a wavelength.**



**In phase:**

$$\Delta d = m\lambda$$

**Out of phase:**

$$\Delta d = \left(m + \frac{1}{2}\right)\lambda$$

- Where  $m = 0, 1, 2, 3, 4, \dots$

#### EXAMPLE 16.10

#### Interference of sound from two speakers

Susan stands directly in front of two speakers that are in line with each other. The farther speaker is 6.0 m from her; the closer speaker is 5.0 m away. The speakers are connected to the same 680 Hz sound source, and Susan hears the sound loud and clear. The frequency of the source is slowly increased until, at some point, Susan can no longer hear it. What is the frequency when this cancellation occurs? Assume that the speed of sound in air is 340 m/s.

- When looking at spherical waves, the path difference is going to be related the radius from the observation point to each individual speaker.
- $\Delta r$  is the path-length difference

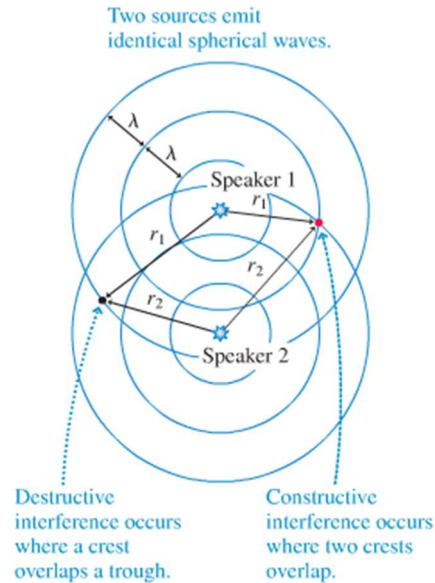
$$\Delta r = |r_2 - r_1|$$

- Constructive interference occurs when:

$$\Delta r = m\lambda$$

- Destructive interference occurs when:

$$\Delta r = \left(m + \frac{1}{2}\right)\lambda$$



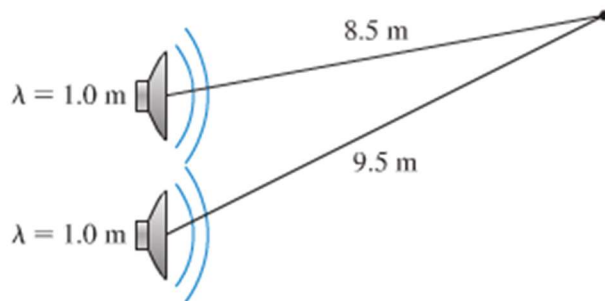
#### EXAMPLE 16.11

#### Is the sound loud or quiet?

Two speakers are 3.0 m apart and play identical tones of frequency 170 Hz. Sam stands directly in front of one speaker at a distance of 4.0 m. Is this a loud spot or a quiet spot? Assume that the speed of sound in air is 340 m/s.

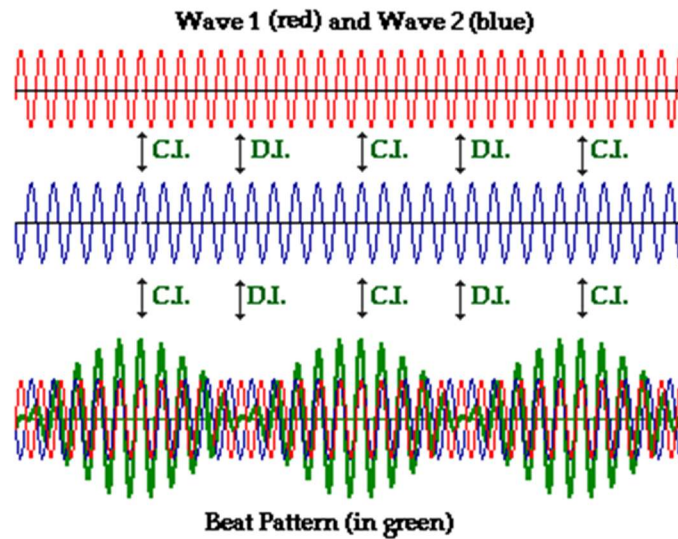
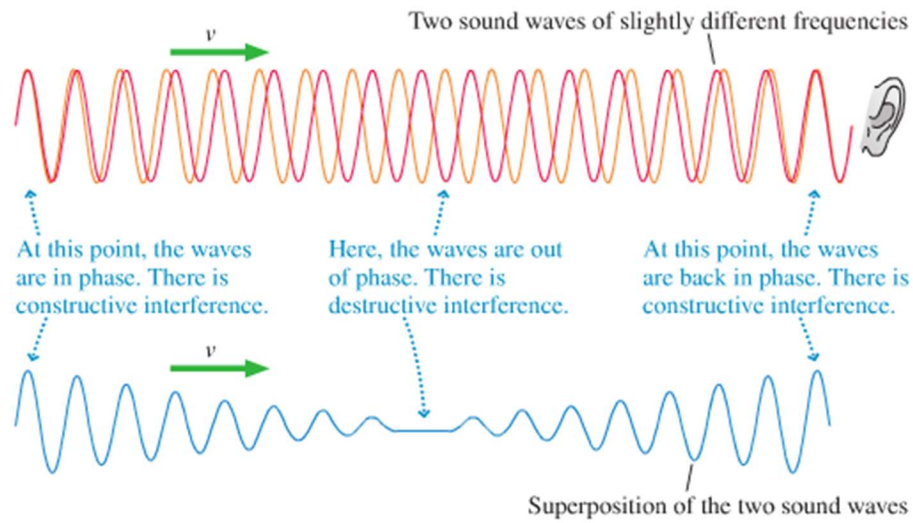
#### STOP TO THINK 16.7

These speakers emit identical sound waves with a wavelength of 1.0 m. At the point indicated, is the interference constructive, destructive, or something in between?



#### BEATS

- When two waves travel towards your ear and have the same amplitude but *slightly* different frequencies, they combine in a manner that alternates between constructive and destructive interference.
  - As it hits your ear, you hear an alternating intensity. (i.e., the sound intensity fluctuates between high and low).
  - Called **beats**.



- The human ear is only capable of determining beats with frequency differences of 7 Hz or below.
- The beat frequency is the difference between the two beats

$$f_{beat} = |f_1 - f_2|$$

- This is how musicians tune instruments. If there's a beat, the instrument is not fully tuned.