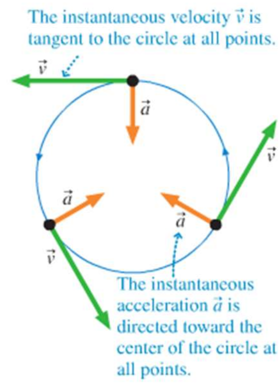
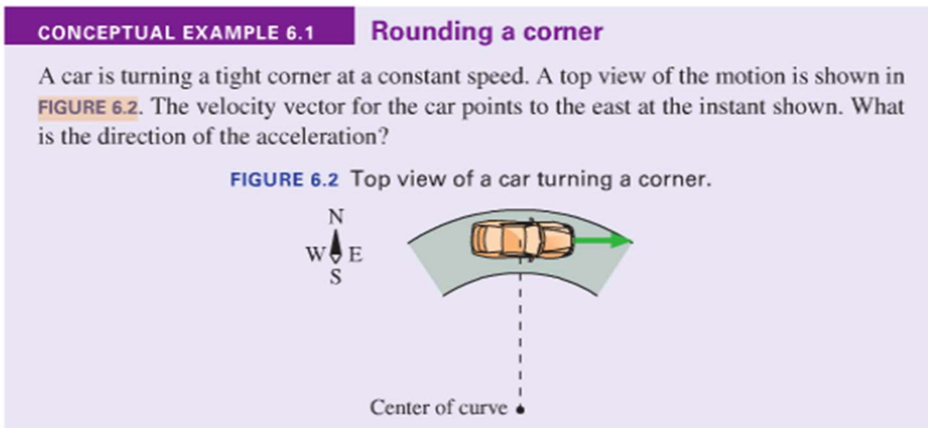
UNIFORM CIRCULAR MOTION

- **Uniform circular motion**→ going in a circle at a constant speed.
- UCM requires an acceleration directed towards the center of the circle.
- *Speed* is constant. Velocity is *not*.
    - ➢ Constantly changing dir'n
    - ➢ There is acceleration at every point in the motion
    - ➢ $\vec{a}$ points towards the center
    - ➢ Called **centripetal acceleration**



$$a = \frac{v^2}{r}$$


CONCEPTUAL EXAMPLE 6.1 Rounding a corner

A car is turning a tight corner at a constant speed. A top view of the motion is shown in FIGURE 6.2. The velocity vector for the car points to the east at the instant shown. What is the direction of the acceleration?

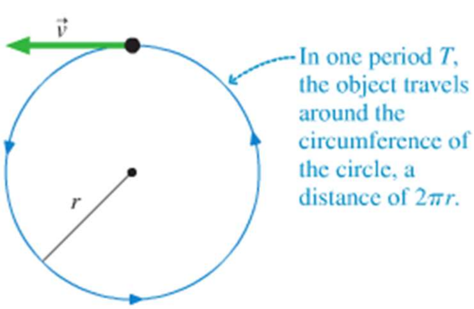
FIGURE 6.2 Top view of a car turning a corner.

Note: you can have circular motion without completing a full circle, as seen in the above example.

PERIOD, FREQUENCY AND SPEED

- **Period** (T)→ the time it takes an object to complete one revolution (rev). Units: s
- **Frequency** (f)→ number of revolutions per second. Units: Hertz (Hz) or s^{-1}

$$f = \frac{1}{T}$$

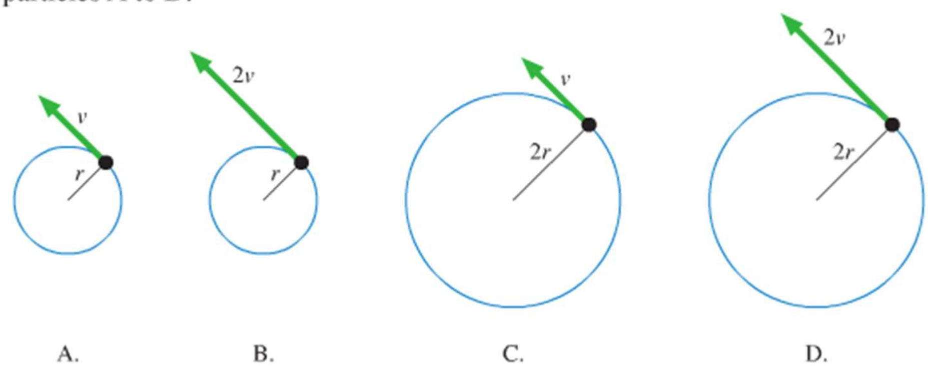


$$v = \frac{2\pi r}{T}$$

EXAMPLE 6.2 **Spinning some tunes**

An audio CD has a diameter of 120 mm and spins at up to 540 rpm. When a CD is spinning at its maximum rate, how much time is required for one revolution? If a speck of dust rides on the outside edge of the disk, how fast is it moving? What is the acceleration?

STOP TO THINK 6.1 Rank in order, from largest to smallest, the period of the motion of particles A to D.

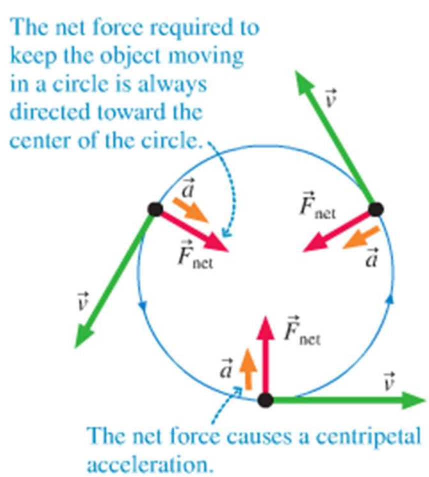


DYNAMICS OF UNIFORM CIRCULAR MOTION

- Newton's 2nd Law→ a net force causes acceleration.

$$\sum F = \frac{mv^2}{r}$$

- This net force is called **centripetal force**.
    - Not a new force. It's a net force. The product of already familiar forces (tension, friction, etc.)



CONCEPTUAL EXAMPLE 6.4 Forces on a car, part I

Engineers design curves on roads to be segments of circles. They also design dips and peaks in roads to be segments of circles with a radius that depends on expected speeds and other factors. A car is moving at a constant speed and goes into a dip in the road. At the very bottom of the dip, is the normal force of the road on the car greater than, less than, or equal to the car's weight?

CONCEPTUAL EXAMPLE 6.5 Forces on a car, part II

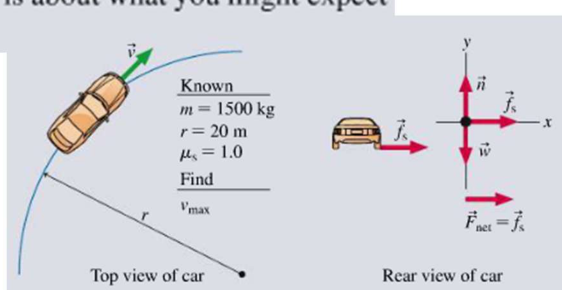
A car is turning a corner at a constant speed, following a segment of a circle. What force provides the necessary centripetal acceleration?

EXAMPLE 6.6 Analyzing the motion of a cart

An energetic father places his 20 kg child on a 5.0 kg cart to which a 2.0-m-long rope is attached. He then holds the end of the rope and spins the cart and child around in a circle, keeping the rope parallel to the ground. If the tension in the rope is 100 N, how much time does it take for the cart to make one rotation?

EXAMPLE 6.7 Finding the maximum speed to turn

What is the maximum speed with which a 1500 kg car can make a turn around a curve of radius 20 m on a level (unbanked) road without sliding? (This radius turn is about what you might expect at a major intersection in a city.)



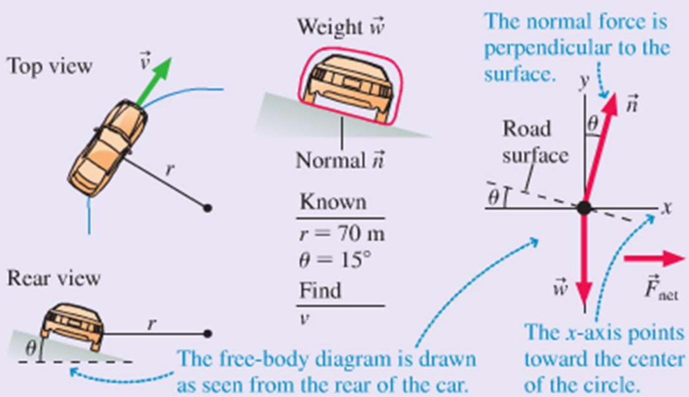
- The maximum speed will depend on the road conditions (μ).
- How do racecars maximize speed around turns?
  - Wings on Formula 1 or Indy cars push air upwards, which provides an extra downward force on the car (Newton's 3rd Law).
  - Banking curves means that some of the Normal force is directed towards the center of the circle.

EXAMPLE 6.8 **Finding speed on a banked turn**

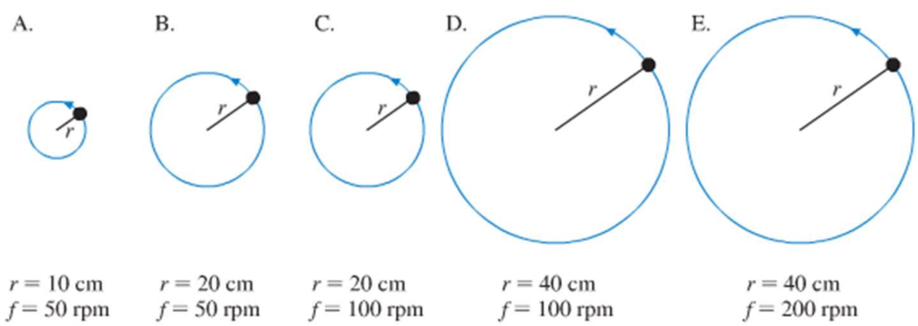
A curve on a racetrack of radius 70 m is banked at a 15° angle. At what speed can a car take this curve without assistance from friction?

PREPARE After drawing the pictorial representation in FIGURE 6.11, we use the force identification diagram to find that, given that there

FIGURE 6.11 Visual overview for the car on a banked turn.

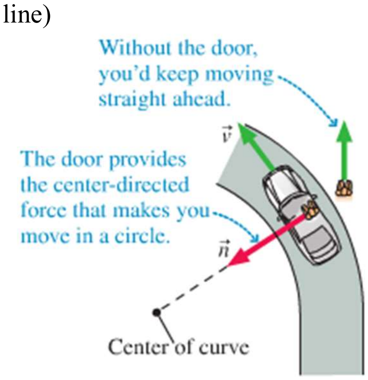


STOP TO THINK 6.2 A block on a string spins in a horizontal circle on a frictionless table. Rank in order, from largest to smallest, the tensions T_A to T_E acting on the blocks A to E.

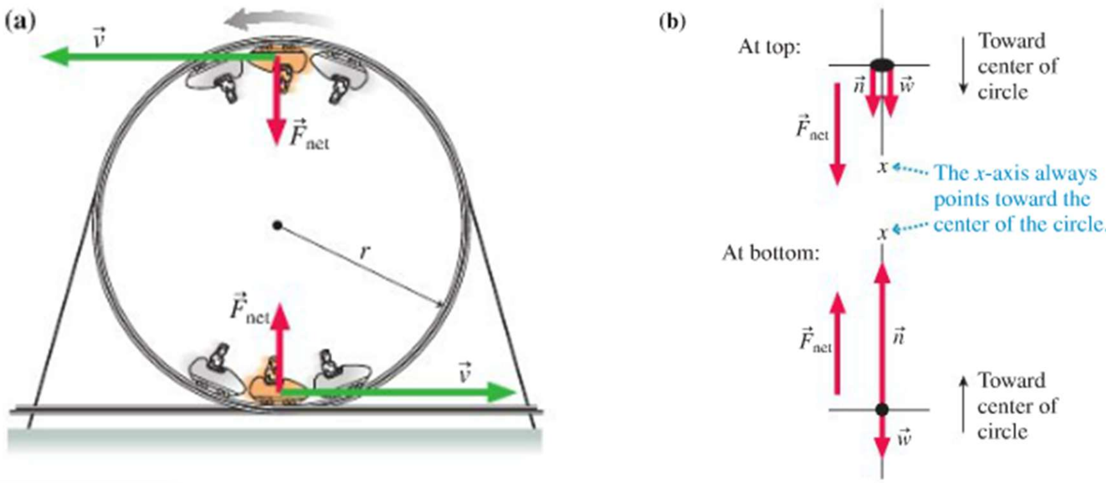


APPARENT FORCES IN CIRCULAR MOTION

- **Centrifugal force→** a "force" that *seems* to push an object to the outside of a circle.
  - Not a real force.
  - It's the result of inertia (wanting to move in a straight line)
  - A centrifugal force will **never** appear on a free body diagram nor be included in Newton's laws.



- Apparent weight also plays a factor in circular motion.
    - The force you *feel*, your apparent weight, is the magnitude of the contact force that supports you.



EXAMPLE 6.9 **How slow can you go?**

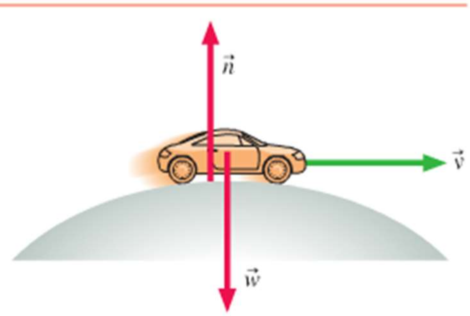
A motorcyclist in the Globe of Death, pictured here, rides in a 2.2-m-radius vertical loop. To keep control of the bike, the rider wants the normal force on his tires at the top of the loop to equal or exceed his and the bike's combined weight. What is the minimum speed at which the rider can take the loop?

EXAMPLE 6.10 **Analyzing the ultracentrifuge**

An 18-cm-diameter ultracentrifuge produces an extraordinarily large centripetal acceleration of 250,000g, where g is the free-fall acceleration due to gravity. What is its frequency in rpm? What is the apparent weight of a sample with a mass of 0.0030 kg?

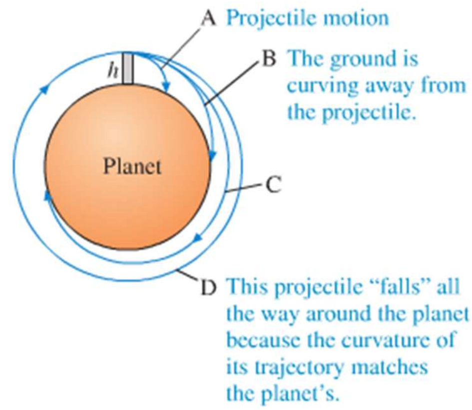
STOP TO THINK 6.3 A car is rolling over the top of a hill at constant speed v. At this instant,

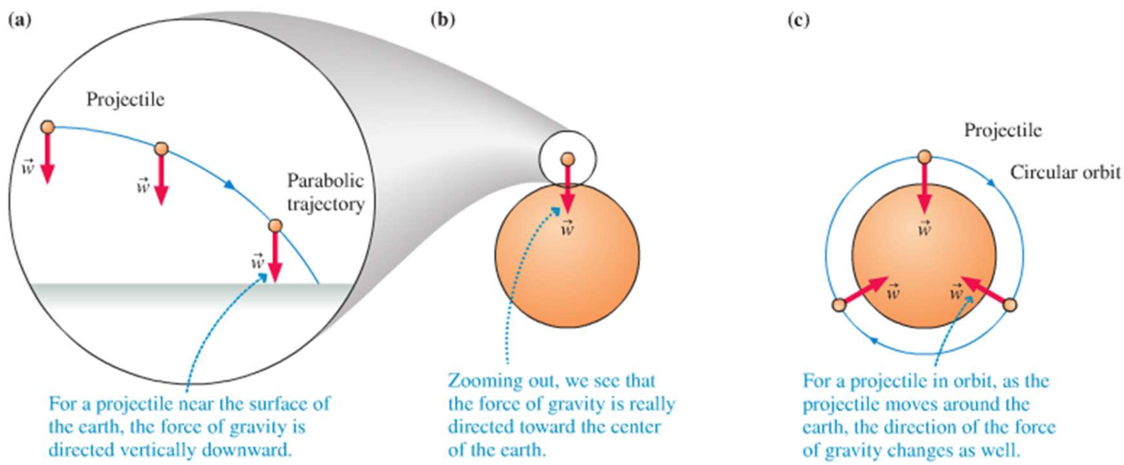
A. $n > w$.
B. $n < w$.
C. $n = w$.
D. We can't tell about n without knowing v.



CIRCULAR ORBITS AND WEIGHTLESSNESS

- Think back to Newton's Mountain.
    - A projectile with a high enough initial velocity will fall, but never reach the ground.
    - It is in **orbit**
    - **An orbiting projectile is in free fall.**



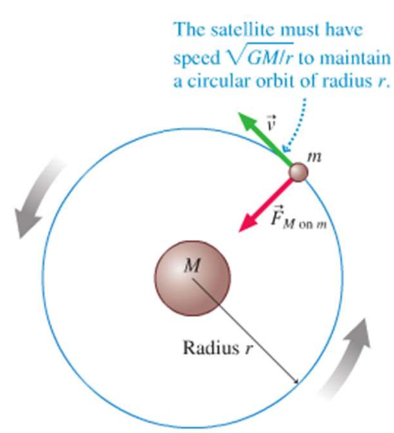


- **Satellite**→ something orbiting a planetary body.
    - Man-made: satellites, ISS
    - Natural: Moon

STOP TO THINK 6.4 A satellite is in a low earth orbit. Which of the following changes would increase the orbital period?

A. Increasing the mass of the satellite.
B. Increasing the height of the satellite about the surface.
C. Increasing the value of g.

- Using the Law of Universal Gravitation, we can derive a formula for the speed required for a satellite to maintain a circular orbit



$$v = \sqrt{\frac{GM}{r}}$$

- Note: the orbital speed doesn't depend on mass. Just like with projectiles and free-fall motion.
- If the satellite is going any faster than that orbital speed, it will result in an elliptical orbit.
- If we combine the orbital speed equation and the speed equation derived earlier, we can find the period of a satellite.

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

- Note: the M in the above equation is the mass of the object at the center of the orbit.

EXAMPLE 6.15 Locating a geostationary satellite

Communication satellites appear to "hover" over one point on the earth's equator. A satellite that appears to remain stationary as the earth rotates is said to be in a *geostationary orbit*. What is the radius of the orbit of such a satellite?

GRAVITY ON LARGE SCALE

- Gravity→ weak but long-ranged
- Why doesn't gravity pull all of the stars together?
  - Galaxy doesn't rotate like a fixed object.
  - Different stars have different orbital periods.
  - Our neighboring stars today, could be on the other side of the galaxy from us in the future.
  - Our solar system (app. 5 billion years old) has only orbited galactic center 20 times.

STOP TO THINK 6.6 Each year, the moon gets a little bit farther away from the earth, increasing the radius of its orbit. How does this change affect the length of a month?

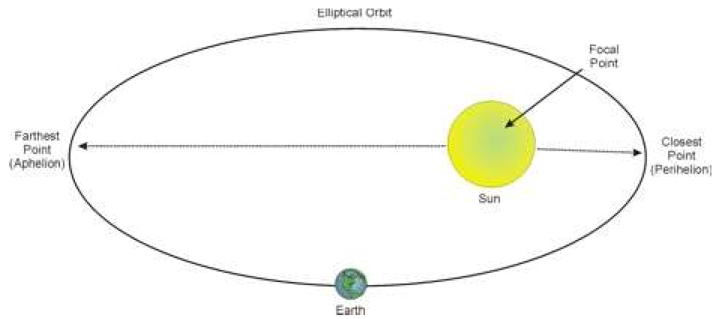
A. A month gets longer.
B. A month gets shorter.
C. The length of a month stays the same.

INTEGRATED EXAMPLE 6.16 A hunter and his sling

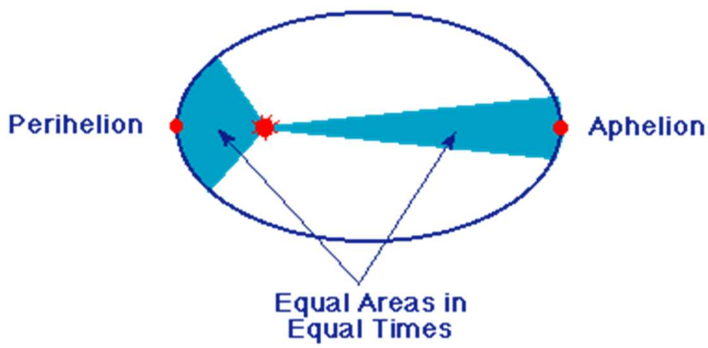
A Stone Age hunter stands on a cliff overlooking a flat plain. He places a 1.0 kg rock in a sling, ties the sling to a 1.0-m-long vine, then swings the rock in a horizontal circle around his head. The plane of the motion is 25 m above the plain below. The tension in the vine increases as the rock goes faster and faster. Suddenly, just as the tension reaches 200 N, the vine snaps. If the rock is moving toward the cliff at this instant, how far out on the plain (from the base of the cliff) will it land?

KEPLER'S LAWS OF PLANETARY MOTION

- **Kepler's First Law→** the planets move in elliptical orbits with the Sun at one focus.



- **Kepler's Second Law→** The straight line joining the Sun and any planet sweeps out equal areas in equal intervals of time.



- **Kepler's Third Law→** gives a formula for the period of an orbit. (see equation on previous page)

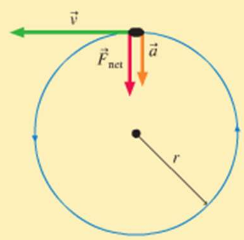
SUMMARY

Goal: To learn about motion in a circle, including orbital motion under the influence of a gravitational force.

GENERAL PRINCIPLES

Uniform Circular Motion

An object moving in a circular path is in uniform circular motion if v is constant.



- The speed is constant, but the direction of motion is constantly changing.
- The **centripetal acceleration** is directed toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

- This acceleration requires a net force directed toward the center of the circle. Newton's second law for circular motion is

$$\vec{F}_{net} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$

Universal Gravitation

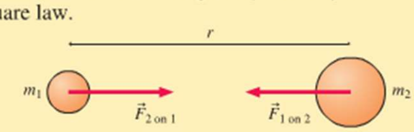
Two objects with masses m_1 and m_2 that are distance r apart exert attractive gravitational forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

where the gravitational constant is

$$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$$

This is **Newton's law of gravity.** Gravity is an inverse-square law.



IMPORTANT CONCEPTS

Describing circular motion

For an object moving in a circle of radius r at a constant speed v:

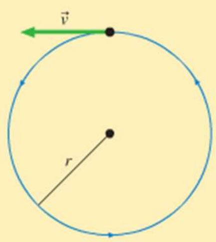
- The **period** T is the time to go once around the circle.

  T = time for one revolution

- The **frequency** f is defined as the number of revolutions per second. It is defined in terms of the period:

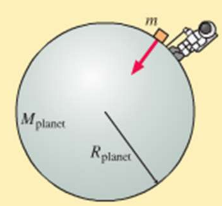
$$f = \frac{1}{T}$$

- The frequency and period are related to the speed and the radius: $v = 2\pi fr = \frac{2\pi r}{T}$



Planetary gravity

The gravitational attraction between a planet and a mass on the surface depends on the two masses and the distance to the center of the planet.



$$F_{\text{planet on } m} = \frac{GM_{planet}m}{R_{planet}^2}$$

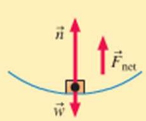
We can use this to define a value of the free-fall acceleration at the surface of a planet:

$$g_{planet} = \frac{GM_{planet}}{R_{planet}^2}$$

APPLICATIONS

Apparent weight and weightlessness

Circular motion requires a net force pointing to the center. The apparent weight $w_{app} = n$ is usually not the same as the true weight w. n must be > 0 for the object to be in contact with a surface.



In orbital motion, the net force is provided by gravity. An astronaut and his spacecraft are both in free fall, so he feels weightless.

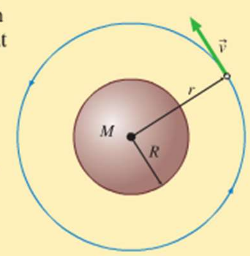
Orbital motion

A **satellite** in a circular orbit of radius r around an object of mass M moves at a speed v given by

$$v = \sqrt{\frac{GM}{r}}$$

The period and radius are related as follows:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$



The speed of a satellite in a low orbit is

$$v = \sqrt{gr}$$

The orbital period is

$$T = 2\pi\sqrt{\frac{r}{g}}$$