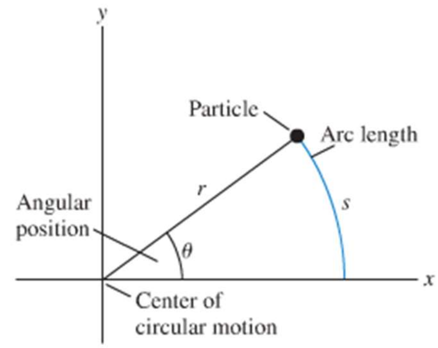
ROTATIONAL MOTION

ROTATIONAL KINEMATICS

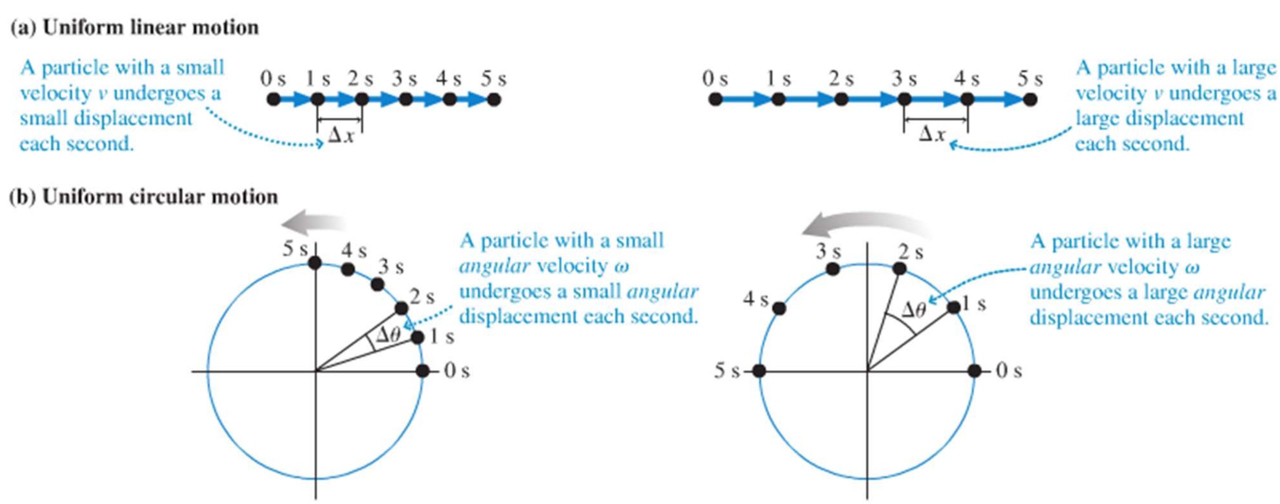
- **Rotational Motion** → the motion of objects that spin about an axis
  - Ex: a fan
  - Particle travels with a fixed radius (r)
  - **Angular position** → ϴ indicates the position of the particle
  - When angular position is counterclockwise from the + x-axis, it's positive. When clockwise, it's negative.
  - **Arc length** (s) → distance travelled along a circular path.
  - Angular position is measured in radians.

$$\theta = \frac{s}{r}$$



- 1 revolution = 360º = 2π rad
- **Angular velocity** → the measure of how much angular displacement an object goes through per second.
  - Measured in rad/s

$$\omega = \frac{\Delta\theta}{\Delta t}$$



- **Angular velocity is constant for an object moving in *uniform* circular motion.**

EXAMPLE 7.1 Comparing angular velocities

Find the angular velocities of the two particles in Figure 7.2b.

PREPARE For uniform circular motion, we can use any angular displacement $\Delta\theta$, as long as we use the corresponding time interval Δt. For each particle, we'll choose the angular displacement corresponding to the motion from $t = 0$ s to $t = 5$ s.

EXAMPLE 7.2 Kinematics at the roulette wheel

A small steel ball rolls counterclockwise around the inside of a 30.0-cm-diameter roulette wheel, like the one shown in the chapter preview. The ball completes exactly 2 rev in 1.20 s.

a. What is the ball's angular velocity?
b. What is the ball's angular position at $t = 2.00$ s? Assume $\theta_i = 0$.

- **Period** → the amount of time it takes to complete one full cycle.
- An object moving in a circle goes 2π rad in a period, T.

$$\omega = (2\pi\, rad)f$$

EXAMPLE 7.3 Rotations in a car engine

The crankshaft in your car engine is turning at 3000 rpm. What is the shaft's angular speed?

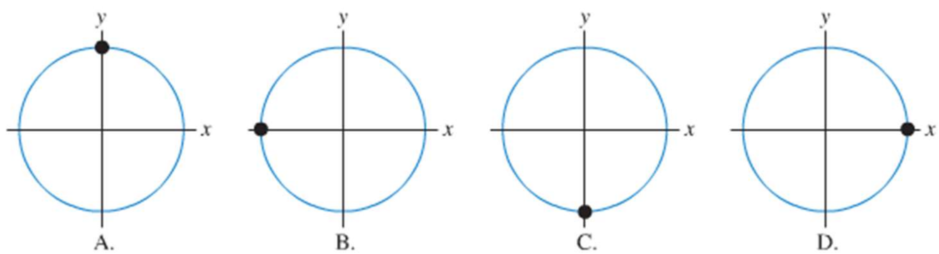
- Different points on a rotating object have different speeds. They have the *same* angular velocity, but their linear speeds change based off distance from the center. But **every point has the same angular velocity.**

$$v = \omega r$$

EXAMPLE 7.5 Finding the speed at two points on a CD

The diameter of an audio compact disk is 12.0 cm. When the disk is spinning at its maximum rate of 540 rpm, what is the speed of a point (a) at a distance 3.0 cm from the center and (b) at the outside edge of the disk, 6.0 cm from the center?

STOP TO THINK 7.1 Which particle has angular position $5\pi/2$?



- **Angular acceleration** → not uniform circular motion. Angular velocity changes over time.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

SYNTHESIS 7.1 Linear and circular motion

The variables and equations for linear motion have analogs for circular motion.

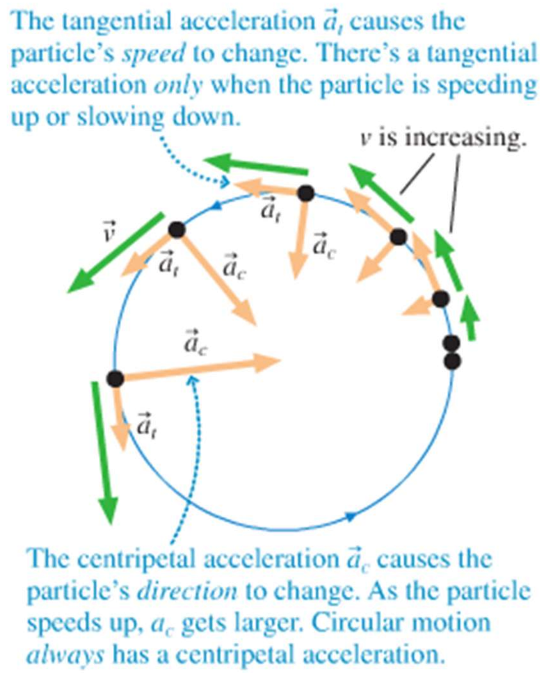
		Linear motion	Circular motion	
Variables	Position (m)	x	θ	Angle (rad)
	Velocity (m/s)	$v_x = \dfrac{\Delta x}{\Delta t}$	$\omega = \dfrac{\Delta\theta}{\Delta t}$	Angular velocity (rad/s)
	Acceleration (m/s^2)	$a_x = \dfrac{\Delta v_x}{\Delta t}$	$\alpha = \dfrac{\Delta\omega}{\Delta t}$	Angular acceleration (rad/s^2)
Equations	Constant velocity	$\Delta x = v\Delta t$	$\Delta\theta = \omega\Delta t$	Constant angular velocity
	Constant acceleration	$\Delta v = a\Delta t$	$\Delta\omega = \alpha\Delta t$	Constant angular acceleration
		$\Delta x = v\Delta t + \frac{1}{2}a(\Delta t)^2$	$\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	

EXAMPLE 7.6 Spinning up a computer disk

The disk in a computer disk drive spins up from rest to a final angular speed of 5400 rpm in 2.00 s. What is the angular acceleration of the disk? At the end of 2.00 s, how many revolutions has the disk made?

- Centripetal acceleration is directed towards the center. But if an object moving in a circle is increasing angular velocity, it experiences **tangential acceleration**.

(b) Nonuniform circular motion



$$a_t = \alpha r$$

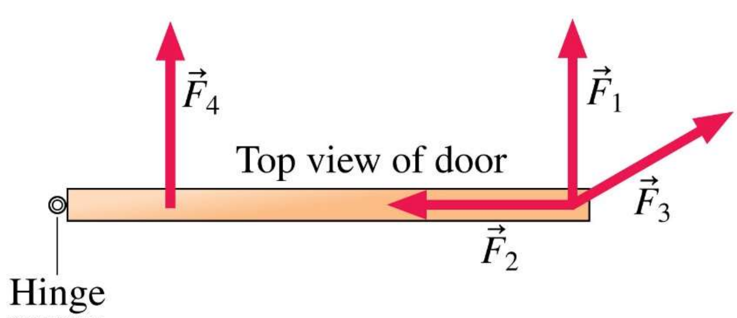
STOP TO THINK 7.2 A ball on the end of a string swings in a horizontal circle once every second. State whether the magnitude of each of the following quantities is zero, constant (but not zero), or changing.

a. Velocity
b. Angular velocity
c. Centripetal acceleration
d. Angular acceleration
e. Tangential acceleration

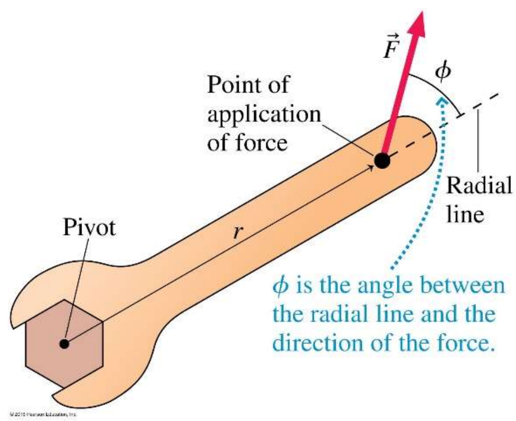
- Recap of all our accelerations acting on a rotating object:
  - ➢ Centripetal acceleration → exists if the object is changing direction (not speed).
  - ➢ Tangential acceleration → exists if the object is changing both direction and linear/tangential speed.
  - ➢ Angular acceleration → exists if there is a change in angular speed.

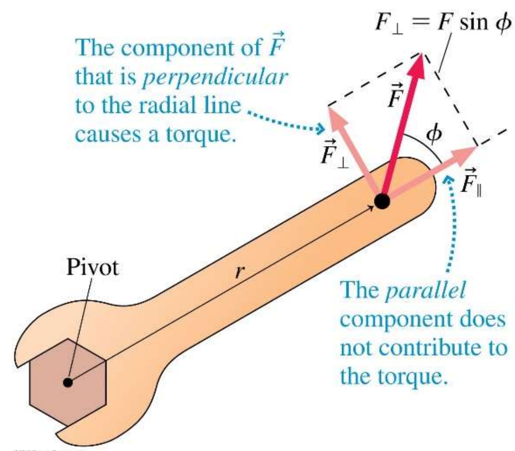
TORQUE

- Which force will most easily open the door pictured below?



- The ability of a force to cause a rotation depends on 3 factors:
  - ➢ The magnitude of the force.
  - ➢ The distance (r) from the pivot to the point at which the force is applied.
  - ➢ The angle at which the force is applied.
- **Torque** → the rotational equivalent of force.
  - ➢ The three factors listed above are the components of torque.
  - ➢ Comes from the Latin *torquere* → "to twist"
- **Radial line**: a line from the pivot point (or fulcrum) to the point at which the force is applied. It extends beyond that point, too.
  - ➢ The angle, phi, is the angle btween the radial line and the force vector.
  - ➢ **Only the perpendicular component of the force has an effect on the object's rotation.**



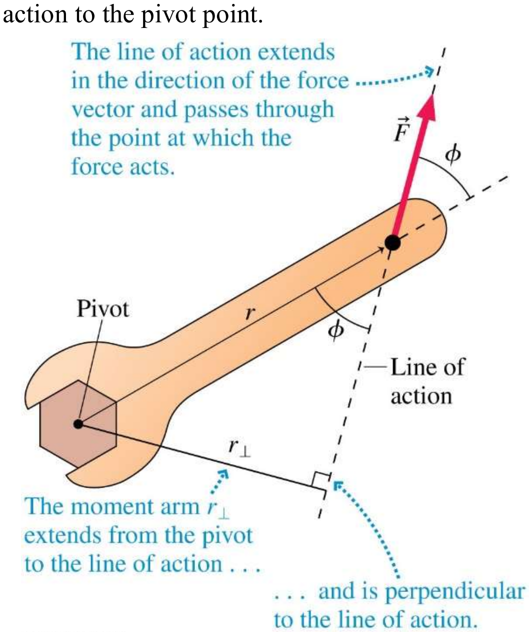


$$\tau = F_\perp r$$

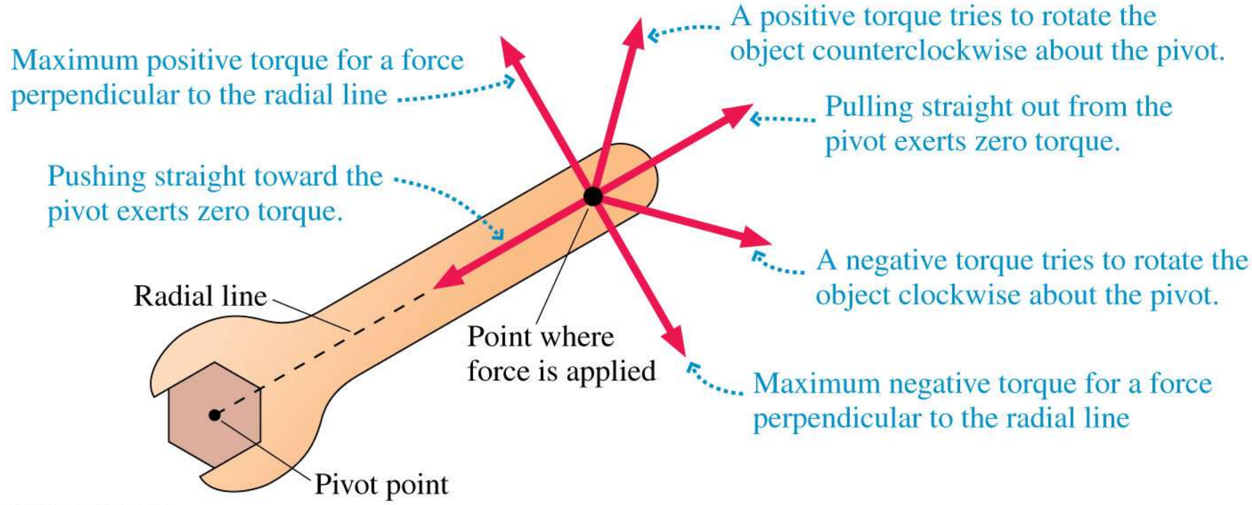
  - ➢ Measured in Newton-meters (N·m)
- Alternate way to find torque:
  - ➢ **Line of action:** the line extending to infinity on either side of the force vector.
  - ➢ **Moment arm:** the perpendicular line from the line of action to the pivot point.

$$\tau = r_\perp F$$

$$\tau = Fr\ sin\phi$$



- **Torque differs from force in a VERY important way.**
  - Torque is measured/calculated about a particular point. Expressing torque without specifying from where is meaningless.
  - In reality, it can be calculate from any point, but in practice we calculate torques from a hinge, pivot or fulcrum.
- Torque is positive when it causes an object to rotate counterclockwise.



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CONCEPTUAL EXAMPLE 7.8 **Starting a bike**

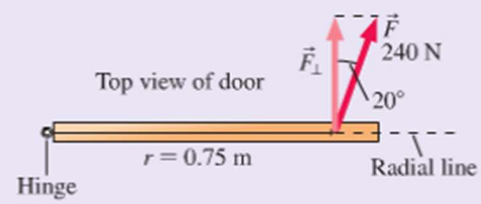
It is hard to get going if you try to start your bike with the pedal at the highest point. Why is this?

EXAMPLE 7.9 **Torque in opening a door**

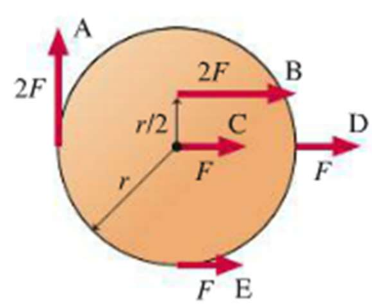
Ryan is trying to open a stuck door. He pushes it at a point 0.75 m from the hinges with a 240 N force directed 20° away from being perpendicular to the door. There's a natural pivot point, the hinges. What torque does Ryan exert? How could he exert more torque?

PREPARE In FIGURE 7.20 the radial line is shown drawn from the pivot—the hinge—through the point at which the force $\vec{F}$ is applied.

FIGURE 7.20 Ryan's force exerts a torque on the door.



STOP TO THINK 7.3 A wheel turns freely on an axle at the center. Given the details noted in Figure 7.21, which one of the forces shown in the figure will provide the largest positive torque on the wheel?



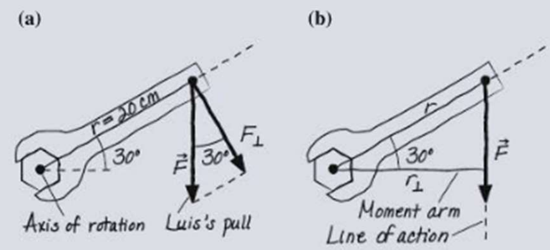
EXAMPLE 7.10 **Calculating the torque on a nut**

Luis uses a 20-cm-long wrench to tighten a nut, turning it clockwise. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?

PREPARE FIGURE 7.22 shows the situation. The two illustrations correspond to two methods of calculating torque, corresponding to Equations 7.10 and 7.11.

SOLVE According to Equation 7.10, the torque can be calculated as $\tau = rF_{\perp}$. From Figure 7.22a, we see that the perpendicular component of the force is

FIGURE 7.22 A wrench being used to turn a nut.

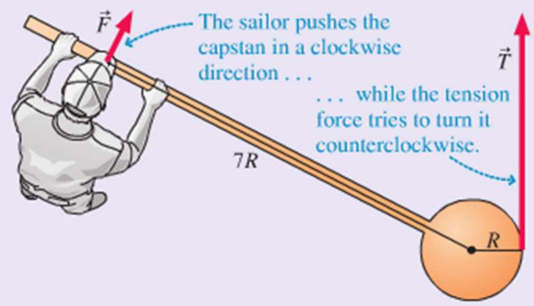


EXAMPLE 7.11 **Force in turning a capstan**

A capstan is a device used on old sailing ships to raise the anchor. A sailor pushes the long lever, turning the capstan and winding up the anchor rope. If the capstan turns at a constant speed, the net torque on it, as we'll learn later in the chapter, is zero.

Suppose the rope tension due to the weight of the anchor is 1500 N. If the distance from the axis to the point on the lever where the sailor pushes is exactly seven times the radius of the capstan around which the rope is wound, with what force must the sailor push if the net torque on the capstan is to be zero?

FIGURE 7.24 Top view of a sailor turning a capstan.



<u>ROTATIONAL EQUILIBRIUM</u>

- **Rotational** Equilibrium → when a pivoted object is balanced horizontally so it doesn't revolve.
- **Law of the Lever** → Unequal forces on a pivoted bar balance each other if

$$\sum \tau = 0$$

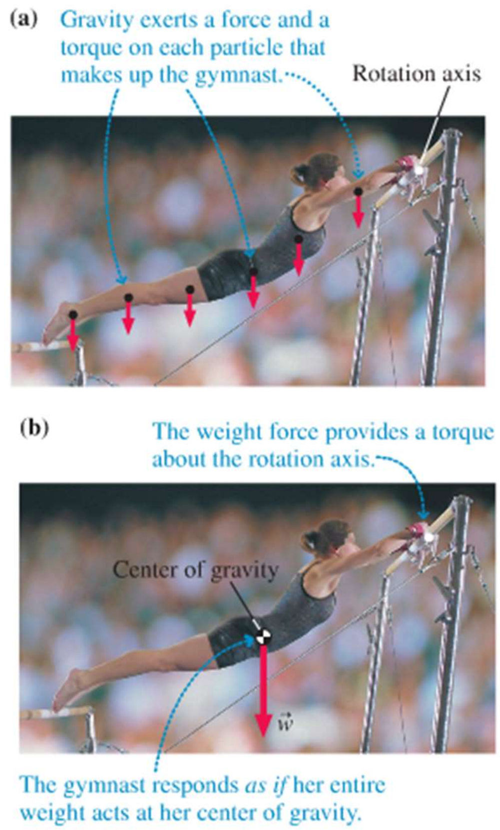
$$F_1 r_1 = F_2 r_2$$

Example:

A boy (mass 30 kg) wishes to play on a centrally pivoted seesaw with his dog Irving (mass 10 kg). When the dog sits 3.0 m from the pivot, where must the boy sit if the 6.5 m long board is to be balanced horizontally?

CENTER OF GRAVITY

- **Center of Gravity** → the point where the total weight (F_g) of an object can be imagined to act.
  - ➢ If something is pivoted about its CoG, there is no torque.

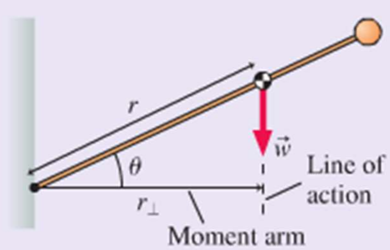


EXAMPLE 7.12 **The torque on a flagpole**

A 3.2 kg flagpole extends from a wall at an angle of 25° from the horizontal. Its center of gravity is 1.6 m from the point where the pole is attached to the wall. What is the gravitational torque on the flagpole about the point of attachment?

PREPARE FIGURE 7.26 shows the situation. For the purpose of calculating torque, we can consider the entire weight of the pole

FIGURE 7.26 Visual overview of the flagpole.



Known
$m = 3.2$ kg
$r = 1.6$ m
$\theta = 25°$

Find
Torque τ

- **Center of Mass** → A point at which we can imagine all of an object's mass to be located
    - ➢ When the gravitational field is uniform over the object (which is usually the case), center of gravity and center of mass are in the same location.

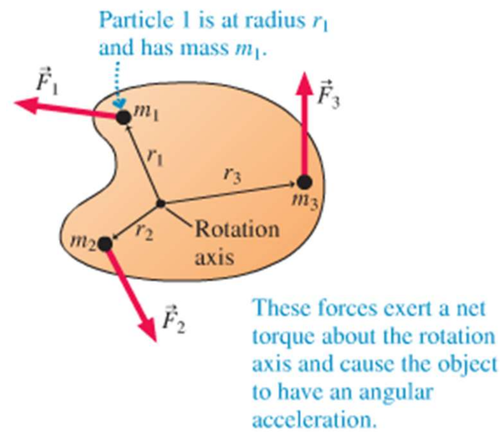
ROTATIONAL INERTIA

- Just like in linear motion, there is inertia.
- Using Newton's 2nd, we can derive:

$$\tau = Fr$$

$$\tau = m\alpha r^2$$

FIGURE 7.34 The forces on a rigid body exert a torque about the rotation axis.



- All particles will have the same angular acceleration on a rigid body, though their masses and radii will differ.
- The **moment of inertia** (I) is the inertial resistance to rotation and depends on the mass and radii of the particles:

$$I = \Sigma mr^2$$

SYNTHESIS 7.2 Linear and rotational dynamics

The variables for linear dynamics have analogs for rotational dynamics. Newton's second law for rotational dynamics is expressed in terms of these variables.

	Linear dynamics	Rotational dynamics
Variables	Net force (N) → $\vec{F}_{net}$	τ_{net} ← Net torque (N · m)
	Mass (kg) → m	I ← Moment of inertia (kg · m^2)
	Acceleration (m/s^2) → $\vec{a}$	α ← Angular acceleration (rad/s^2)
Newton's second law	Acceleration is caused by forces. The larger the mass, the smaller the acceleration. $\vec{a} = \frac{\vec{F}_{net}}{m}$	$\alpha = \frac{\tau_{net}}{I}$ Angular acceleration is caused by torques. The larger the moment of inertia, the smaller the angular acceleration.

- **The more mass and the farther it is from the axis of rotation, the greater will be I, and the greater will be resistance to the change in rotational motion.**

<u>ANGULAR MOMENTUM</u>

$$L = I\omega$$

- Units: kg·m/s

TABLE 9.2 Rotational and linear dynamics

Rotational dynamics	**Linear dynamics**
Torque τ_{net}	Force $\vec{F}_{net}$
Moment of inertia I	Mass m
Angular velocity ω	Velocity $\vec{v}$
Angular momentum $L = I\omega$	Linear momentum $\vec{p} = m\vec{v}$

- **Law of Conservation of Angular Momentum →** in the absence of a net torque, angular momentum will remain constant.

Final (f) and initial (i) moment of inertia and angular velocity of object 1

$$\overbrace{(I_1)_f(\omega_1)_f} + \underbrace{(I_2)_f(\omega_2)_f} + \cdots = \overbrace{(I_1)_i(\omega_1)_i} + \underbrace{(I_2)_i(\omega_2)_i} + \cdots$$

Final (f) and initial (i) moment of inertia and angular velocity of object 2